

Vortex lines in the three-dimensional XY model with random phase shifts

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The stability of the ordered phase of the three-dimensional XY model with random phase shifts is studied by considering the roughening of a single stretched vortex line due to disorder. It is shown that the vortex line may be described by a directed polymer Hamiltonian with an effective random potential that is long-range correlated. A Flory argument estimates the roughness exponent to $\zeta=3/4$ and the energy fluctuation exponent to $\omega=1/2$, thus fulfilling the scaling relation $\omega=2\zeta-1$. The Schwartz-Edwards method as well as a numerical integration of the corresponding Burger's equation confirms this result. Since $\zeta<1$, the ordered phase of the original XY model is stable. [S0163-1829(96)00945-9]

I. INTRODUCTION

Systems with a continuous symmetry of the order parameter are particularly susceptible to the influence of frozen-in disorder.¹ In this paper we investigate the stability of the ordered phase of the three-dimensional XY model with random phase shifts with respect to topological defects. Strictly speaking, in order to distinguish between a quantum spin model, in which only the X and Y components of the spins interact (the true XY model), and the classical two-component spin model we consider here, it would be more appropriate to refer to our model as a planar rotator model. However, since this distinction is not often followed in the literature, we keep the notation XY model throughout this paper.

The Hamiltonian of this model may be written in the form

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - A_{ij}), \quad (1)$$

where J is an effective coupling, the phase variables $\theta_i \in [0, 2\pi]$ are placed on a three-dimensional (3D) hypercubic lattice, and the sum runs over all nearest-neighbor pairs $\langle ij \rangle$. The variables A_{ij} are quenched random phase shifts (or random gauges) on the bonds connecting nearest neighbors. For simplicity we assume that the A_{ij} on different bonds are uncorrelated and Gaussian distributed with mean zero and variance σ . Below it will be useful to go over to the continuum description in which A_{ij} is replaced by the field $\vec{A}(\vec{r})$.

Model (1) describes XY magnets with a random Dzyaloshinskii-Moriya interaction.² Other realizations of this model are 3D-Josephson-junction arrays with positional disorder³ and vortex glasses.⁴

In the case of the so-called gauge glass model⁴ one assumes A_{ij} to be uniformly distributed between 0 and 2π but we expect that our model with Gaussian disorder is equivalent to the gauge glass model when σ is large enough.

In $d=2$ dimensions the model (1) shows a low-temperature weak-disorder phase with quasi-long-range order (QLRO) and, for $\sigma=0$, a vortex-driven Kosterlitz-Thouless-like transition^{5,2,6} to the disordered phase. A finite amount of disorder shifts the transition to lower temperatures and reduces the universal jump in the spin-spin correlation exponent from $\eta=1/4$ for $\sigma=0$ to $\eta=1/16$ at $\sigma_c=\pi/8$. As was shown recently,^{6,7} this transition is not reentrant, contrary to earlier findings,² but in agreement with results of Nishimori for a model with slightly different correlations of the random phase shifts.⁸

In $d=3$ dimensions, it is easy to show that spin-wave excitations, which couple only to $\vec{\nabla} \cdot \vec{A}$, do not destroy true long range order at all temperatures and disorder strengths. To address the question of whether this picture is qualitatively changed even for weak disorder and low temperatures by allowing vortex configurations we consider a *single* stretched vortex line in the presence of the disorder field \vec{A} . Such a vortex line can be forced into the system e.g., by appropriate boundary conditions. As long as the vortex line remains self-affine, i.e., as long as its typical transverse displacement u scales as L^ζ (where L denotes the length of the line) with a roughness exponent $\zeta<1$, which implies (for

weak disorder and low T) a finite line tension, we conclude that the ordered phase remains stable. It turns out that this problem can be mapped approximatively onto a directed polymer problem with long-range correlated disorder. We find both from analytical and numerical calculations that the roughness exponent is indeed about 0.75, such that the ordered phase remains stable. This is in agreement with results of Nishimori⁸ for a slightly different probability weight for the disorder.

Although these arguments are not rigorous and strictly speaking further instabilities cannot be excluded, experience with other random systems, in particular the random field Ising model, suggests⁹ that it is the instability of large-scale topological defects which lead to a destruction of the ordered phase below a certain critical dimensions.

II. SPIN WAVES AND VORTICES

In order to separate between spin-wave and vortex degrees of freedom we start from the continuum description of model (1):

$$\mathcal{H} = \frac{J}{2} \int d^3r |\vec{\nabla} \theta(\vec{r}) - \vec{A}(\vec{r})|^2. \quad (2)$$

The quenched vector field $\vec{A}(\vec{r})$ is assumed to be Gaussian distributed with zero average and correlations ($\alpha, \beta = 1, 2, 3$)

$$\langle A_\alpha(\vec{r}) A_\beta(\vec{r}') \rangle = \sigma \delta_{\alpha\beta} \delta(\vec{r} - \vec{r}'), \quad (3)$$

where $\langle \dots \rangle$ denotes the disorder average.

The original model (1) is periodic in $\theta_i - \theta_j$ with the periodicity 2π . In order to preserve this periodicity in the continuum version (2) we have to allow for singularities along which $\nabla \theta$ jumps by 2π . These singular surfaces are bounded by vortex lines l which are characterized by their topological charge m_l and their position vector $\vec{R}_l(s)$. It is convenient to decompose the $\vec{\theta}$ field into a spin-wave part $\vec{\nabla} \theta_{\text{sw}}$ and a vortex part $\vec{\nabla} \theta_v$. The spin-wave part is vortex free. The vortex part of the vector field $\vec{\nabla} \theta_v$ is defined by the saddle point equations

$$\vec{\nabla} \cdot [\vec{\nabla} \theta_v - \vec{A}(\vec{r})] = 0 \quad (4)$$

and

$$\vec{\nabla} \times (\vec{\nabla} \theta_v) = 2\pi \vec{m}(\vec{r}), \quad (5)$$

where $\vec{m}(\vec{r})$ denotes the vortex density field, which is non-zero only along the singular lines $\vec{R}_l(s)$,

$$\vec{m}(\vec{r}) = \sum_l m_l \int ds \frac{d\vec{R}_l(s)}{ds} \delta^{(3)}(\vec{r} - \vec{R}_l(s)). \quad (6)$$

Equation (5) and (6) can be solved easily by introducing a vector potential $\vec{a}(\vec{r})$, $\vec{\nabla} \times \vec{a}(\vec{r}) = \vec{\nabla} \theta_v - \vec{A}(\vec{r})$. The solution is

$$\begin{aligned} \vec{\nabla} \theta_v(\vec{r}) = & \int d^3r' G(\vec{r} - \vec{r}') \{ -\vec{\nabla}' [\vec{\nabla}' \cdot \vec{A}(\vec{r}')] \\ & + 2\pi \vec{\nabla}' \times \vec{m}(\vec{r}') \}, \end{aligned} \quad (7)$$

with $\vec{\nabla} \cdot \vec{m} = 0$. Here $\vec{\nabla}'$ denotes a derivative with respect to the primed variable \vec{r}' and \times represents a vector product. The Green function $G(\vec{r})$ satisfies $\nabla^2 G(\vec{r}) = -\delta(\vec{r})$. In three dimensions $G(\vec{r})$ takes the form $G(\vec{r}) = (4\pi r)^{-1}$.

Representing $\theta(\vec{r})$ as a sum of a vortex part $\theta_v(\vec{r})$ and a spin-wave part $\theta_{\text{sw}}(\vec{r})$ one can show that Hamiltonian (2) is decomposed into two independent spin-wave and vortex parts. In what follows we will be interested only in the vortex part \mathcal{H}_v which may be written in the form

$$\mathcal{H}_v = \mathcal{H}_{vv} + \mathcal{H}_{vd},$$

$$\mathcal{H}_{vv} = \frac{J\pi}{2} \int \int d^3r d^3r' \frac{\vec{m}(\vec{r}) \cdot \vec{m}(\vec{r}')}{|\vec{r} - \vec{r}'|}, \quad (8)$$

$$\mathcal{H}_{vd} = \frac{J}{2} \int \int d^3r d^3r' \frac{\vec{m}(\vec{r}) \cdot [\vec{\nabla}' \times \vec{A}(\vec{r}')] }{|\vec{r} - \vec{r}'|}.$$

The lattice version of such a vortex Hamiltonian for gauge glass has been derived recently.¹⁰ Obviously, \mathcal{H}_{vv} and \mathcal{H}_{vd} correspond to the vortex-vortex and vortex-disorder interactions, respectively. As in $d=2$ dimensions, the vortices couple only to the $\vec{\nabla} \times \vec{A}(\vec{r}) = 2\pi \vec{Q}(\vec{r})$, which can be considered as a quenched random vector charge field with $\vec{\nabla} \cdot \vec{Q} = 0$.

As follows from Eq. (3), $\langle \vec{Q}(\vec{r}) \rangle = 0$ and

$$\langle Q_\alpha(\vec{r}) Q_\beta(\vec{r}') \rangle = \frac{\sigma}{4\pi} [\partial_\alpha \partial_\beta - \delta_{\alpha\beta} \nabla^2] \delta(\vec{r} - \vec{r}'). \quad (9)$$

Equations (8) and (9) would be the starting point for the statistical treatment of model (1). In the partition function we had to integrate over all possible configurations of vortex loops and vortex lines spanning the system. However, this task is much too difficult and remains to be done even for the pure system.

Instead, we follow here a much more modest approach and consider the case of a very strong dilution of vortex lines. In order to test the stability of the vortex-free state with respect to vortex formation, it is indeed sufficient to consider an isolated large vortex loop. Without disorder, such a vortex loop costs an energy of the order $\sim \Sigma_0 L \ln L$ (see below) if L is the radius of the loop and $\Sigma_0 \sim Jm^2$ denotes the bare line tension of the vortex line. The configurational entropy of the loop is also of the order $L \ln L$ and hence will produce a negative free energy only at sufficiently high temperatures. At these temperature the system is then disordered, and the state of the system is characterized by multiply entangled vortex lines and loops.

However, at low temperatures this mechanism does not work. A possible source for the condensation of vortex loops here is the disorder. To check this possibility, we consider in the next section a single stretched vortex line which we allow to become rough under the influence of the disorder. As long as the typical transverse distortion $u \sim t^\zeta$ of a piece of length t of the vortex line is characterized by a roughness exponent $\zeta < 1$, the contribution $\delta \Sigma_t$ of the disorder to the

total vortex line tension, Σ , from distortions on this length scale is of the order $-\Sigma_0 u^2/t^2 \propto -t^{2(\zeta-1)}$ and hence small for large t .

Summation over the contributions $\delta\Sigma_t$ from all length scales $b^n a$ (a being the microscopic length scale or small-scale lattice cutoff and b the usual renormalization factor) between $n_{\min}=0$ and $n_{\max}=\ln(L/a)/\ln b$ yields a finite value for $\delta\Sigma_{\text{total}}$ which can be made arbitrarily small for decreasing disorder strength. Thus the line tension $\Sigma=\Sigma_0+\delta\Sigma_{\text{total}}$ remains positive and the system is stable with respect to the condensation of vortex lines.

However, for $\zeta=1$ the disorder contributions to the line tension are independent of t and hence the energy per unit length will vanish on a sufficiently large length scale t_c . As a result, vortex loops of size $L>t_c$ will appear spontaneously and hence destroy the ordered phase. In this way we traced back the existence of a vortex-free low-temperature phase to the determination of the roughness exponent ζ of a single stretched vortex line. We expect no difference in the case of a vortex loop as long as t_c is sufficiently large, which can be always achieved for weak disorder.

III. SINGLE-VORTEX-LINE HAMILTONION

We consider in the following only a *single* stretched vortex line with no overhangs, which means that we may set $\vec{R}(s)\rightarrow\vec{R}(t)=(r_1(t), r_2(t), t)$, where $\mathbf{r}(t)=(r_1(t), r_2(t))$ describes the transverse distortion of the vortex line from a straight configuration. Then $\vec{m}(\vec{r})$ may be parametrized as follows:¹¹

$$\vec{m}(\vec{r})=m\int dt\frac{d\vec{R}}{dt}\delta^{(3)}(\vec{r}-\vec{R}(t)),$$

$$\frac{d\vec{R}}{dt}=\left(\frac{\partial r_1}{\partial t}, \frac{\partial r_2}{\partial t}, 1\right). \quad (10)$$

\mathcal{H}_{vv} describes now the elastic self-interaction of the vortex line $\mathcal{H}_{vv}\rightarrow\mathcal{H}_{\text{el}}$:

$$\mathcal{H}_{\text{el}}=\frac{J\pi}{2}m^2\int\int\frac{d\vec{R}(t)d\vec{R}(t')}{|\vec{R}(t)-\vec{R}(t')|},$$

$$d\vec{R}(t)d\vec{R}(t')=\left(\frac{\partial\mathbf{r}}{\partial t}\cdot\frac{\partial\mathbf{r}}{\partial t'}+1\right)dt dt'. \quad (11)$$

Expanding \mathcal{H}_{vv} in Eq. (8) up to quadratic terms in \mathbf{r} and $\partial\mathbf{r}/\partial t$ and omitting an irrelevant constant corresponding to the energy of straight lines one has

$$\mathcal{H}_{\text{el}}=\frac{J\pi}{2}m^2\int\int\frac{dt dt'}{|t-t'|}\left[\frac{\partial\mathbf{r}}{\partial t}\cdot\frac{\partial\mathbf{r}}{\partial t'}-\frac{[\mathbf{r}(t)-\mathbf{r}(t')]^2}{2|t-t'|^2}\right]$$

$$+O(r^4). \quad (12)$$

It is convenient to go over to the Fourier transform $\mathbf{r}(t)=\int_0^1 e^{i\omega t}\mathbf{r}_\omega d\omega/(2\pi)$:

$$\mathcal{H}_{\text{el}}=\frac{Jm^2}{2}\int\frac{d\omega}{(2\pi)}\omega^2 f(a\omega)\mathbf{r}_\omega\cdot\mathbf{r}_{-\omega}, \quad (13)$$

where a denotes a small-scale lattice cutoff and

$$f(\epsilon)=\int_\epsilon^\infty\frac{dx}{x}\left[\left(1+\frac{1}{x^2}\right)\cos x-\frac{1}{x^2}\right]\approx-\ln\epsilon-\frac{3}{4} \quad (14)$$

for $\epsilon\rightarrow 0$.

Next we consider the correlations of the vortex disorder interaction. For this purpose we rewrite \mathcal{H}_{vd} as

$$\mathcal{H}_{vd}=\int d\vec{R}\cdot\vec{V}(\vec{R}), \quad (15)$$

where

$$\vec{V}(\vec{R})=J\pi m\int d^3r\frac{\vec{Q}(\vec{r})}{|\vec{R}-\vec{r}|}. \quad (16)$$

Clearly $\langle\vec{V}\rangle=0$ and

$$\langle V_\alpha(\vec{R})V_\beta(\vec{R}')\rangle=\sigma\frac{\pi J^2 m^2}{2}\frac{1}{|\vec{R}-\vec{R}'|}$$

$$\times\left\{\delta_{\alpha\beta}+\frac{(R_\alpha-R'_\alpha)(R_\beta-R'_\beta)}{(\vec{R}-\vec{R}')^2}\right\}. \quad (17)$$

Since the correlator (17) always appears under the integral over \vec{R} , it is convenient to rewrite the right-hand side of (Eq. 17) as

$$\frac{\pi}{2}\sigma(Jm)^2\left(\frac{\partial^2}{\partial R_\alpha\partial R'_\beta}|\vec{R}-\vec{R}'|+\frac{\delta_{\alpha\beta}}{|\vec{R}-\vec{R}'|}\right).$$

The first contribution in this expression leads to terms in the correlator of \mathcal{H}_{vd} which vanish by assuming periodic boundary conditions. Thus we omit it in the following. For small gradients $|\partial r/\partial t|\ll 1$, \mathcal{H}_{vd} can finally be rewritten as

$$\mathcal{H}_{vd}=\int dt V(\mathbf{r}(t), t), \quad (18)$$

with

$$\langle V(\mathbf{r}, t)V(\mathbf{r}', t')\rangle=\sigma(Jm)^2|(\mathbf{r}-\mathbf{r}')^2+(t-t')^2|^{-1/2}. \quad (19)$$

Thus, the random potential $V(\vec{R})$ which interacts with the vortex line is long-range correlated.

IV. FLORY ARGUMENTS AND BEYOND

In the following we want to estimate the conditions under which the random potential can destabilize the vortex line. We begin with a *straight* line of length t , which has an elastic energy $E_{\text{el}}=(J\pi/2)m^2 t \ln t/a$ as follows easily from Eqs. (11). The typical fluctuations of the disorder energy follows from Eq. (17) as $E_{\text{dis}}=\pm mJ\sqrt{\pi\sigma}(t \ln t/a)^{1/2}$. Since rare fluctuations increase E_{dis} only by a factor of $(\ln t)^{(1/2)}$, the system is always stable with respect to the formation of straight vortex lines.

Next we allow for a displacement $\mathbf{r}(t)$ from the straight configuration such that $|\partial\mathbf{r}/\partial t|\ll 1$. If on the scale t the vor-

tex line is self-affine with $u = \langle \mathbf{r}^2 \rangle^{1/2} \sim t^\zeta$ with $\zeta < 1$, we can use a Flory argument to estimate ζ . The elastic energy is then increased by $\Delta E_{el} \approx (Jm^2 \pi/2) t^{-1} u^2 \ln t/a$ which has to be compared with E_{dis} . This yields

$$u \sim [\sigma t^3 (\ln t/a)^{-1}]^{1/4}, \quad (20)$$

i.e., $\zeta = 3/4$. Although the Flory argument is rather crude, it seems to be safe enough to conclude $\zeta < 1$; i.e., for weak disorder the vortex line is self-affine and hence stable (only $\zeta = 1$ would signal an instability with respect to vortex generation).

To go beyond the Flory argument one usually maps these types of directed polymer problems to Burger's equation with (correlated) noise. However, in the present case this is not possible in the strict sense because of the long-range elastic self-interaction of the vortex line, which leads to the factor $f(a\omega) \approx \ln(1/a\omega)$ in Eq. (14). Since this corresponds to a logarithmic increase of the stiffness of the vortex line with the length scale, we neglect the ω dependence of this factor in the following completely and set equal to f_0 . If then the roughness exponent ζ is less than 1, it will be less than 1 by including $f(a\omega)$. In fact, it is even safe to assume that the value of ζ is unchanged by replacing $f(a\omega)$ by f_0 since for small $a\omega$ $f(a\omega)$ diverges only logarithmically.

With this simplification the free energy $h(\mathbf{r}, t)$ of a vortex line of length t and $\mathbf{r} = \mathbf{r}(t) - \mathbf{r}(0)$ obeys the Burger's equation with noise

$$\frac{\partial}{\partial t} h(\mathbf{r}, t) = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + V(\mathbf{r}, t), \quad (21)$$

where $\nu = T/2Jm^2 f_0$ and $\lambda = 1/Jm^2 f_0$.

In addition to the roughness exponent ζ there is a second exponent ω , which describes the sample to sample fluctuations $\langle h^2(\mathbf{r}, t) \rangle - \langle h(\mathbf{r}, t) \rangle^2 \sim t^{2\omega}$ of the free energy. Since the correlator (19) is nonlocal in time, the validity of the scaling relation $\omega = 2\zeta - 1$ is not guaranteed.¹²

If we give $h(\mathbf{r}, t)$ another interpretation, namely, that of the height profile of a growing interface in a comoving frame, Eq. (21) is known as the Kardar-Parisi-Zhang (KPZ) equation.¹³ The roughness exponent α , the growth exponent β , and the dynamical exponent z of this *surface* are related to the directed polymer exponents by

$$\begin{aligned} \zeta &= \beta/\alpha = 1/z, \\ \omega &= \beta = \alpha/z. \end{aligned} \quad (22)$$

We first attempted to use the standard one-loop renormalization group treatment for the Burger's equation.¹² Unfortunately, this approach does not yield (as happens also for the case of uncorrelated noise in three dimensions) stable fixed points from which one could calculate critical exponents. We study the Burger's equation with the correlated noise [Eq. (21)] therefore here by a method that proved useful for the same problem with uncorrelated noise.¹⁴

Our first step here is to describe our system as a system of dynamical variables affected by noise that is uncorrelated in time. This is necessary in order to go over from the Langevin-like Burger's equation to the Fokker-Planck equation, needed in the method mentioned above.

This step is achieved by considering the V to be also dynamical variables coupled to a noise ξ that is uncorrelated in space and time.

In Fourier components we write Eq. (21) in the form

$$\frac{\partial h_{\mathbf{q}}}{\partial t} = -\nu q^2 h_{\mathbf{q}} - \frac{\lambda}{2\sqrt{\Omega}} \sum_{\mathbf{T}} \mathbf{1} \cdot (\mathbf{q} - \mathbf{1}) h_{\mathbf{T}} h_{\mathbf{q}-\mathbf{T}} - V_{\mathbf{q}}(t), \quad (23)$$

where Ω is the transverse area of the system (to be taken eventually to infinity) and add to it an equation for $V_{\mathbf{q}}(t)$:

$$\frac{\partial V_{\mathbf{q}}}{\partial t} = -|\mathbf{q}| V_{\mathbf{q}} + \xi_{\mathbf{q}}(t), \quad (24)$$

where $\langle \xi_{\mathbf{q}}(t) \rangle = 0$ and $\langle \xi_{\mathbf{q}}(t) \xi_{-\mathbf{q}}(t') \rangle = 2D \delta(t - t')$; $2D$ is short for $4\pi\sigma(Jm^2)$. It is easily verified that $V_{\mathbf{q}}(t)$ has the required correlations given by Eq. (19).

We write now the Fokker-Planck equation for the joint probability density to have a given h and V configuration, $P\{h_{\mathbf{q}}, V_{\mathbf{q}}\}$:

$$\begin{aligned} \frac{\partial P}{\partial t} = \sum_{\mathbf{q}} \left\{ \frac{\partial}{\partial V_{\mathbf{q}}} \left(D \frac{\partial}{\partial V_{\mathbf{q}}} + |\mathbf{q}| V_{\mathbf{q}} \right) \right. \\ \left. + \frac{\partial}{\partial h_{\mathbf{q}}} \left(\nu q^2 h_{\mathbf{q}} + \frac{\lambda}{2\sqrt{\Omega}} \sum_{\mathbf{T}} \mathbf{1} \cdot (\mathbf{q} - \mathbf{1}) h_{\mathbf{T}} h_{\mathbf{q}-\mathbf{T}} + V_{\mathbf{q}} \right) \right\} P. \end{aligned} \quad (25)$$

We will be interested in the steady state averages $\langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle_s$, $\langle h_{\mathbf{q}} V_{-\mathbf{q}} \rangle_s$, and $\langle V_{\mathbf{q}} V_{-\mathbf{q}} \rangle_s$, and in some characteristic frequencies to be defined later. We assume now that the exact values of $\langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle_s$, $\langle h_{\mathbf{q}} V_{-\mathbf{q}} \rangle_s$, and $\langle V_{\mathbf{q}} V_{-\mathbf{q}} \rangle_s$ are known and given by $X_{\mathbf{q}}^{-1}$, $\Delta_{\mathbf{q}}^{-1}$ and $\Lambda_{\mathbf{q}}^{-1}$. In order to calculate such quantities, we need some form of a perturbation expansion in which the Fokker-Planck operator O , acting on P on the right-hand side of Eq. (24), will be broken into two parts: one part O_0 that is simple enough and another part $O - O_0$ that is small enough. We may expect that if we choose O_0 in such a way that it already gives the exact result for the three steady state averages, the corrections to those quantities in perturbation theory are going to be small, giving sense to the whole expansion.

Our choice for O_0 is

$$\begin{aligned} O_0 = \sum_{\mathbf{q}} \omega_{\mathbf{q}}^{(1)} \frac{\partial}{\partial V_{\mathbf{q}}} \left[\Lambda_{\mathbf{q}}^{-1} \frac{\partial}{\partial V_{-\mathbf{q}}} + V_{\mathbf{q}} \right] \\ + \omega_{\mathbf{q}}^{(2)} \frac{\partial}{\partial h_{\mathbf{q}}} \left[X_{\mathbf{q}}^{-1} \frac{\partial}{\partial h_{-\mathbf{q}}} + h_{\mathbf{q}} \right] \\ + [\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}}^{(2)}] \left[\Delta_{\mathbf{q}}^{-1} \frac{\partial}{\partial h_{\mathbf{q}}} \frac{\partial}{\partial V_{-\mathbf{q}}} \right]. \end{aligned} \quad (26)$$

It is easily verified that regardless of the choice of the $\omega_{\mathbf{q}}$'s, O_0 produces $\langle h_{\mathbf{q}} h_{-\mathbf{q}} \rangle_s = X_{\mathbf{q}}^{-1}$, $\langle h_{\mathbf{q}} V_{-\mathbf{q}} \rangle_s = \Delta_{\mathbf{q}}^{-1}$, and $\langle V_{\mathbf{q}} V_{-\mathbf{q}} \rangle_s = \Lambda_{\mathbf{q}}^{-1}$.

A direct inspection of Eq. (24) shows that the form of the V -dependent part of O is identical to the form of the corresponding part of O_0 . Therefore we must identify $\Lambda_{\mathbf{q}}^{-1} = D/|\mathbf{q}|$ and $\omega_{\mathbf{q}}^{(1)} = |\mathbf{q}|$.

Next we calculate in perturbation theory $\langle h_{\mathbf{q}}h_{-\mathbf{q}} \rangle_s$ and $\langle h_{\mathbf{q}}V_{-\mathbf{q}} \rangle_s$. We find

$$\langle h_{\mathbf{q}}h_{-\mathbf{q}} \rangle_s = X_{\mathbf{q}}^{-1} + C_1 \{X_{\mathbf{1}}^{-1}, \Delta_{\mathbf{1}}^{-1}, \omega_{\mathbf{1}}^{(2)}\}, \quad (27)$$

$$\langle h_{\mathbf{q}}V_{-\mathbf{q}} \rangle_s = \Delta_{\mathbf{q}}^{-1} + C_2 \{X_{\mathbf{1}}^{-1}, \Delta_{\mathbf{1}}^{-1}, \omega_{\mathbf{1}}^{(2)}\}.$$

Since $X_{\mathbf{q}}^{-1}$ and $\Delta_{\mathbf{q}}^{-1}$ are assumed to be exact, we get now, for a given set $\{\omega_{\mathbf{q}}^{(2)}\}$, two equations $C_1 = C_2 = 0$ expressing the fact that the perturbation expansion does not change the zero-order value. Actually it could be expected that for any choice of positive $\omega_{\mathbf{q}}^{(2)}$'s, the above equations for the $X_{\mathbf{q}}$'s and $\Delta_{\mathbf{q}}$'s ($C_1 = C_2 = 0$) would give the exact result. It is clear however, that even for statics, we may use the choice of the $\omega_{\mathbf{q}}^{(2)}$'s in order to control the smallness of the additional terms and thus we will be able to obtain our $X_{\mathbf{q}}$'s and $\Delta_{\mathbf{q}}$'s from *low-order* perturbation. Assuming that such a choice is possible we write C_1 and C_2 to second order to obtain the two equations

$$\begin{aligned} 0 = & -[\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}}^{(2)}]\Delta_{\mathbf{q}}^{-1} + \frac{[\nu q^2 - \omega_{\mathbf{q}}^{(2)}]^2}{[\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}}^{(2)}]}\Delta_{\mathbf{q}}^{-1} \\ & + \frac{[\nu q^2 - \omega_{\mathbf{q}}^{(2)}]D}{|\mathbf{q}|[\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}}^{(2)}]} - \frac{D}{|\mathbf{q}|} + \frac{\lambda^2}{(2\pi)^2} \\ & \times \left\{ \int d^2l \frac{\mathbf{1} \cdot (\mathbf{q} - \mathbf{l})}{[\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}-\mathbf{l}}^{(2)} + \omega_{\mathbf{l}}^{(2)}]} \mathbf{q} \cdot (-\mathbf{l}) X_{\mathbf{l}}^{-1} \right\} \Delta_{\mathbf{q}}^{-1} \end{aligned} \quad (28)$$

and

$$\begin{aligned} 0 = & - \left[2\nu q^2 - \frac{(\nu q^2)^2}{\omega_{\mathbf{q}}^{(2)}} \right] X_{\mathbf{q}}^{-1} + [\nu q^2 - \omega_{\mathbf{q}}^{(2)}] \\ & \times \left[\frac{1}{\omega_{\mathbf{q}}^{(2)}} + \frac{1}{\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}}^{(2)}} \right] \Delta_{\mathbf{q}}^{-1} + \frac{D}{|\mathbf{q}|[\omega_{\mathbf{q}}^{(1)} + \omega_{\mathbf{q}}^{(2)}]} \\ & + \frac{\lambda^2}{2(2\pi)^2} \int d^2l \frac{\mathbf{1} \cdot (\mathbf{q} - \mathbf{l})}{[\omega_{\mathbf{l}}^{(2)} + \omega_{\mathbf{q}-\mathbf{l}}^{(2)} + \omega_{\mathbf{q}}^{(2)}]} \mathbf{l} \cdot (\mathbf{q} - \mathbf{l}) X_{\mathbf{l}}^{-1} X_{\mathbf{q}-\mathbf{l}}^{-1} \\ & + 2\mathbf{q} \cdot (\mathbf{l} - \mathbf{q}) X_{\mathbf{q}-\mathbf{l}}^{-1} X_{\mathbf{q}}^{-1}. \end{aligned} \quad (29)$$

We fix now $\omega_{\mathbf{q}}^{(2)}$ to be the characteristic frequency associated with the correlations of $h_{\mathbf{q}}$ at different times:

$$\omega_{\mathbf{q}}^{(2)} = \frac{\langle h_{\mathbf{q}}h_{-\mathbf{q}} \rangle_s}{\int_0^\infty \langle h_{-\mathbf{q}}(\mathbf{0})h_{\mathbf{q}}(t) \rangle dt}, \quad (30)$$

where $\langle A(0)B(t) \rangle$ is defined for $t > 0$ to be

$$\begin{aligned} \langle A(0)B(t) \rangle = & \int A\{h'_{\mathbf{q}}, V'_{\mathbf{q}}\} P_s\{h'_{\mathbf{q}}, V'_{\mathbf{q}}\} P\{h'_{\mathbf{q}}, V'_{\mathbf{q}}; h_{\mathbf{q}}, V_{\mathbf{q}}; t\} \\ & \times B\{h_{\mathbf{q}}, V_{\mathbf{q}}\} Dh'_{\mathbf{q}} DV'_{\mathbf{q}} Dh_{\mathbf{q}} DV_{\mathbf{q}}. \end{aligned} \quad (31)$$

P_s is the static (steady state) distribution and $P\{h'_{\mathbf{q}}, V'_{\mathbf{q}}; h_{\mathbf{q}}, V_{\mathbf{q}}, t\}$ is the solution of the Fokker-Planck equation for initial conditions of absolute certainty that the value of $h_{\mathbf{q}}$ is $h'_{\mathbf{q}}$ and $V_{\mathbf{q}}$ is $V'_{\mathbf{q}}$ [$\prod_{\mathbf{q}} \delta(h_{\mathbf{q}} - h'_{\mathbf{q}}) \delta(V_{\mathbf{q}} - V'_{\mathbf{q}})$ is the initial distribution]. Our lowest-order Fokker-Planck op-

erator O_0 is thus chosen also to give the correct characteristic frequency. We calculate now the characteristic frequency to second order and demand again that the correction to the zero-order term be zero. This yields

$$\begin{aligned} 0 = & \left[2 - \frac{\nu \mathbf{q}^2}{\omega_{\mathbf{q}}^{(2)}} \right] [\omega_{\mathbf{q}}^{(2)} - \nu \mathbf{q}^2] X_{\mathbf{q}}^{-1} \\ & + \frac{\lambda^2}{2(2\pi)^2} \int d^2l \frac{\mathbf{1} \cdot (\mathbf{q} - \mathbf{l})}{[\omega_{\mathbf{l}}^{(2)} + \omega_{\mathbf{q}-\mathbf{l}}^{(2)}]} \\ & \times 2\mathbf{q} \cdot (\mathbf{l} - \mathbf{q}) X_{\mathbf{l}}^{-1} X_{\mathbf{q}}^{-1} + \frac{\lambda^2}{2(2\pi)^2} \int d^2l \\ & \times \frac{\omega_{\mathbf{q}}^{(2)} \mathbf{l} \cdot (\mathbf{q} - \mathbf{l})}{[\omega_{\mathbf{l}}^{(2)} + \omega_{\mathbf{q}-\mathbf{l}}^{(2)}][\omega_{\mathbf{l}}^{(2)} + \omega_{\mathbf{q}-\mathbf{l}}^{(2)} + \omega_{\mathbf{q}}^{(2)}]} \cdot [2\mathbf{q} \cdot (\mathbf{l} - \mathbf{q}) X_{\mathbf{l}}^{-1} X_{\mathbf{q}}^{-1} \\ & + \mathbf{l} \cdot (\mathbf{q} - \mathbf{l}) X_{\mathbf{l}}^{-1} X_{\mathbf{q}-\mathbf{l}}^{-1}] - \frac{\omega_{\mathbf{q}}^{(2)} \Delta_{\mathbf{q}}^{-1}}{\omega_{\mathbf{q}}^{(1)}}. \end{aligned} \quad (32)$$

Apart from the last term, Eq. (32) is identical to the corresponding equation in Ref. 14.

We assume now that for small q

$$\begin{aligned} \omega_{\mathbf{q}}^{(2)} & \propto q^z + \text{corrections}, \\ X_{\mathbf{q}} & \propto q^{\Gamma_1} + \text{corrections}, \\ \Delta_{\mathbf{q}} & \propto q^{\Gamma_2} + \text{corrections}. \end{aligned} \quad (33)$$

The exponents z and Γ_1 are related to the exponents ζ and ω by the relations $\zeta = 1/z$ and $\omega = (\Gamma_1 - 2)/2z$. We solve Eqs. (28), (29), and (32) to leading order and obtain the following results. The scaling relation

$$z = \frac{6 - \Gamma_1}{2} \quad (34)$$

is obeyed. [The last term in Eq. (32) is negligible compared to the dominant terms for small q]. This is the familiar scaling relation

$$\omega = 2\zeta - 1. \quad (35)$$

Equations (28) and (29) allow now for a simple power counting solution (in contrast to Ref. 13 where a power counting solution was impossible).

We obtain

$$\Gamma_1 = \frac{10}{3}, \quad z = \frac{4}{3}, \quad \Gamma_2 = 2. \quad (36)$$

It is interesting to note that the result for Γ_2 is not affected by λ being not equal to zero and this is because the last term in Eq. (28) is negligible, for small q 's, compared to the dominant terms. Consideration of higher-order terms in the expansion (in particular, the most dominant q dependence in each order) seems to suggest that this result will still hold to any order of perturbation theory. In terms of the more familiar directed polymer exponents we find

$$\zeta = \frac{3}{4}, \quad \omega = \frac{1}{2}, \quad (37)$$

so that the Flory result still holds.

V. NUMERICAL RESULTS

In this section we will obtain the roughness exponent for our model by integrating the (2+1)-dimensional KPZ equation (21) numerically. The simplest quantity to investigate is the surface width

$$W = \langle \overline{h^2} - \overline{h}^2 \rangle^{1/2}, \quad (38)$$

where the overbar and angular brackets $\langle \dots \rangle$ denote a spatial and noise averages, respectively. One expects for W_L the scaling form¹⁵

$$W = L^\alpha f(t/L^z), \quad (39)$$

where $f(x) \sim x^\beta$ with $\beta = \alpha/z$ for $x \ll 1$ and $f(x) \sim \text{const}$ for $x \gg 1$; β , α , and z are the growth, roughness, and dynamic exponent of the interface, respectively; cf. Eqs. (22).

In analogy to Eq. (39) the height-height correlation function

$$C_L(\vec{r}, t) = \langle [h(\vec{r} + \vec{r}', t + t') - h(\vec{r}', t')]^2 \rangle^{1/2} \quad (40)$$

should scale as

$$C_L(r, t) = r^\alpha \tilde{g}(r/t^{1/z}, L/t^{1/z}). \quad (41)$$

We do not take the scaling relation

$$\alpha + z = 2, \quad (42)$$

which is equivalent to Eq. (35), for granted in our investigation. Thus, to obtain a full set of the critical exponents we have to calculate at least two of them.

Different theoretical approaches^{12,16,17} lead to different dependences of the critical exponents on ρ in (1+1) dimensions. To check theoretical predictions the numerical simulations have been carried out for the KPZ equation,¹⁸ the ballistic deposition,^{19,18,20} the directed polymer,¹⁸ and the solid-on-solid model.²⁰ There is still a controversy between the simulation results. For example, the results of Amar *et al.*,²⁰ obtained for the ballistic deposition and restricted solid-on-solid models agree with the prediction of Medina *et al.*,¹² but conflict with the prediction of Zhang.¹⁶ Numerical studies¹⁸ of the effect of the long-range spatially correlated noise on the KPZ equation and the related directed polymer problem, on the other hand, give a good agreement with the prediction of Hentschel and Family.¹⁷ Temporally correlated noise in the absence of spatial noise correlations has also been investigated numerically²¹ for the (1+1)-dimensional case.

Theoretical results for (2+1)-dimensional models with correlated noise are still lacking. Meakin and Jullien^{19,22} introduced a hopping model of ballistic deposition, in which particles were deposited on the growing surface following a Levy flight distribution such that the distance x (along the surface) from the previous site was calculated as $x = r^{-1/f}$ where r is a random number between zero and 1. Equating the exponent f to 2 as in Eq. (19) led to results¹⁹ for the exponents α and β which were roughly in agreement with the prediction of Medina *et al.* for (1+1) dimensions. A weak dependence of these exponents on f has been found in the (2+1)-dimensional model.²²

In the simulations of Meakin and Jullien the link between the deposition process and the noise was not obvious. Thus the scaling behavior in (2+1)-dimensional interface growth and the directed polymer model with spatially correlated noise is at the present time unclear. We consider this problem solving the KPZ equation numerically. The spatial derivatives in Eq. (21) are discretized using standard forward-backward differences on a hypercubic grid with a lattice constant Δx . Equation (21) is integrated by the Euler algorithm with time increments Δt . This procedure is nowadays standard for the numerical integration of stochastic differential equations, which is more complicated than that of deterministic differential equations for which higher-order methods are better suited (we refer to Ref. 23 for details). Denoting the grid points by \vec{n} and the basic vectors characterizing the surface by $\vec{e}_1, \dots, \vec{e}_d$ we arrive at the discretized equation²⁴

$$\begin{aligned} \tilde{h}(\vec{n}, \vec{\tau} + \delta \vec{\tau}) = & \tilde{h}(\vec{n}, \vec{\tau}) + \frac{\Delta \vec{\tau}}{\Delta \vec{x}^2} \sum_{i=1}^d \{ [\tilde{h}(\vec{n} + \vec{e}_i, \vec{\tau}) - 2\tilde{h}(\vec{n}, \vec{\tau}) \\ & + \tilde{h}(\vec{n} - \vec{e}_i, \vec{\tau})] + \frac{1}{8} [\tilde{h}(\vec{n} + \vec{e}_i, \vec{\tau}) \\ & - \tilde{h}(\vec{n} - \vec{e}_i, \vec{\tau})]^2 \} + \sqrt{3\Delta \vec{\tau}} \eta(\vec{n}, \vec{\tau}). \end{aligned} \quad (43)$$

Here one uses dimensionless quantities $\tilde{h} = h/h_0$, $\vec{x} = x/x_0$, and $\vec{t} = t/t_0$ where the natural units are given by

$$h_0 = \frac{\nu}{\lambda}, \quad t_0 = \frac{\nu^2}{\sigma^2 \lambda^2}, \quad x_0 = \sqrt{\frac{\nu^3}{\sigma^2 \lambda^2}}, \quad \sigma^2 = 2D/\Delta x^d. \quad (44)$$

Similar to $V(\vec{r}, t)$ in the Fourier space the renormalized noise $\eta(\vec{n}, \vec{\tau})$ has the correlation

$$\langle \eta(\vec{k}, \omega) \eta(\vec{k}', \omega') \rangle \sim \frac{\delta(\vec{k} + \vec{k}') \delta(\omega + \omega')}{k^2 + \omega^2}. \quad (45)$$

To create the spatially correlated noise we follow Peng *et al.*¹⁸ We first generate a standard white (or Gaussian uncorrelated) noise $\eta_0(\vec{n}, \vec{\tau})$, and then carry out the Fourier transformation for spatial and temporal variables to obtain $\eta_0(\vec{k}, \omega)$. We define

$$\eta(\vec{q}, \omega) = (k^2 + \omega^2)^{-1/2} \eta_0(\vec{k}, \omega). \quad (46)$$

The noise $\eta(\vec{n}, \vec{\tau})$ is obtained by Fourier transforming $\eta(\vec{k}, \omega)$ back into the real space. It is easy to check that $\eta(\vec{k}, \omega)$ obtained by this way satisfies Eq. (45).

In our simulations we chose Δx , ν , and σ to be the same as in Ref. 24, namely, $\Delta x = 1$, $\nu = 0.5$, and $\sigma = 0.1$. To be sure that we are in a strong-coupling regime we chose $\lambda = \sqrt{600}$. The data we show in the following are averaged over 10 000 samples (i.e., different noise realizations) for the smallest system size ($L = 16$) to 50 samples for the largest system size ($L = 256$).

First we determine the exponent β and the dynamical exponent z from simulations of large system sizes on short time scales; i.e., we focus on a regime where the correlation length ξ is still small compared to the system size L :

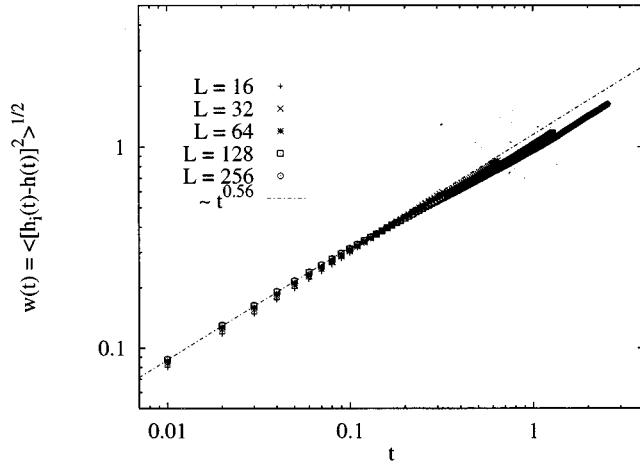


FIG. 1. Log-log plot of the width $W_L(t)$ vs t for various system sizes L . For smaller times the data are roughly independent of system size with a slight tendency to a smaller slope for increasing L . For larger times (i.e., when $t \sim L^z$) significant finite-size effects appear (i.e., the scaling function \tilde{w} in Eq. (48) is no longer constant). Therefore we put more weight on the data points at smaller times in order to determine β , which yields $\beta = 0.56 \pm 0.02$ shown as the dotted line.

$$\xi \propto t^{1/z} \ll L. \quad (47)$$

In this regime finite-size effects should be negligible, which we had to check explicitly by analyzing different system sizes since we do not know z *a priori*. For the time dependence of the roughness we expect the scaling form (39) to hold, which can be cast into the form

$$W_L(t) = t^\beta \tilde{w}(t/L^z), \quad (48)$$

with $\tilde{w}(x) = \text{const}$ for $x \rightarrow 0$ and $\tilde{w}(x) \sim x^{-1}$ for $x \rightarrow \infty$. For short times $t \ll L^z$ one therefore expects $W_L(t) \propto t^\beta$. Figure 1 shows the time dependence of the width $W_L(t)$ for various system sizes L at short-time scales. We find

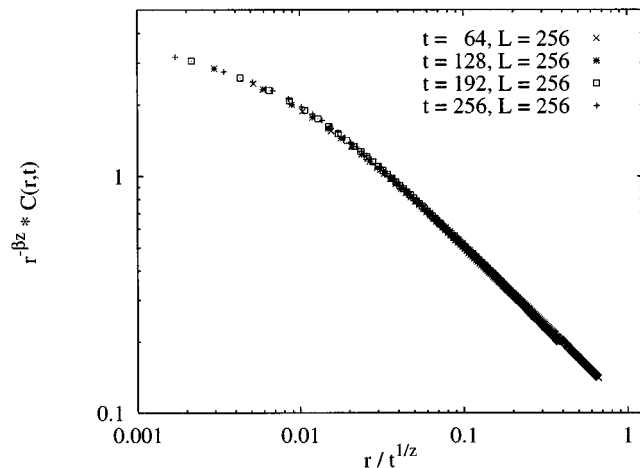


FIG. 2. Scaling plot for the height-height correlation $C(r,t)$ function. We fixed β to be 0.56, as determined before Eq. (49), and varied z in order to achieve the best data collapse, which yields $z = 1.25 \pm 0.05$.

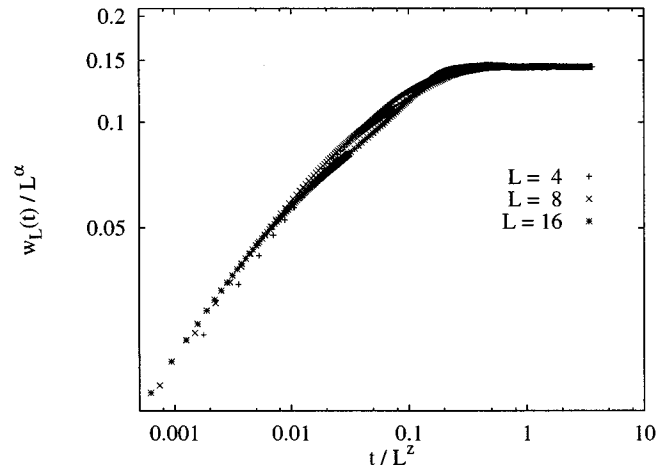


FIG. 3. Scaling plot of $W_L(t)$ according to the scaling form (48). The choice $\alpha = 0.71 \pm 0.01$ and $z = 1.25 \pm 0.03$ yields the best data collapse, which is shown.

$$\beta = 0.56 \pm 0.02. \quad (49)$$

This value is much larger than $\beta \approx 0.240$ for the three-dimensional KPZ equation with the white noise,²⁴ as we expect it to be since correlated noise always increases β .

In order to obtain the dynamical exponent z from the short-time behavior we note that for $t \ll L^z$ the height-height correlation function $C(r,t)$, Eq. (40) should have the scaling form

$$C(r,t) = r^{\beta z} g(r/t^{1/z}), \quad (50)$$

where we denoted $\lim_{(tL^{-z}) \rightarrow 0} \tilde{g}(r/t^{1/z}, L/t^{1/z}) = g(r/t^{1/z})$ and used $\alpha = \beta z$.

If we take $\beta = 0.56$ as determined before, we are left with one fitting parameter by which we should achieve a data collapse when plotting $r^{-\beta z} C(r,t)$ versus $r/t^{1/z}$. In Fig. 2 we show the result of this procedure, which yields

$$z = 1.25 \pm 0.05. \quad (51)$$

Up to now we have restricted ourselves to a regime where finite-size effects are negligible. In order to get an independent estimate for the exponents reported above we performed also a simulation for much longer times and small enough system size so that the correlation length becomes indeed comparable to the system size (i.e., $t \sim L^z$). Using the finite-size scaling form (39) we determine the exponents α and z . In Fig. 3 we show a scaling plot of $W_L(t)$ which yields

$$\alpha = 0.71 \pm 0.01 \quad (52)$$

and $z = 1.25 \pm 0.03$, from which one concludes $\beta = \alpha/z = 0.56 \pm 0.03$. These values agree very well with the values reported in Eqs. (49) and (51) for the short-time simulations.

Using scaling relation (22) we have

$$\omega = 0.56 \pm 0.02 \quad \text{and} \quad \zeta = 0.78 \pm 0.04 \quad (53)$$

for the vortex lines in our model. The value of ζ is larger than the corresponding value $\zeta \approx 0.62$ for the directed polymer paths with uncorrelated noise in three dimensions. Our numerical result is compatible within the error bars with the

crude estimate $\zeta = 3/4$ obtained by the Flory argument and by the more elaborate calculation in the last section. The numerical value of ω is slightly larger than the calculated $\omega = 1/2$.

Note that within the error bars relation (42) is still valid even in the presence of the temporal correlation of noise. This relation may hold true only for some subset of all possible correlators (45). At present we have to leave this question open.

VI. SUMMARY AND CONCLUSION

In the present paper we have considered the stability of the ordered phase of the XY model with random phase shifts. After decomposing the Hamiltonian into a spin-wave and a vortex part, we have considered in particular the roughening of a single stretched vortex line due to the disorder. It turned out that the effective random potential acting on the vortex line is long-range correlated. Using a Flory argument and the Schwartz-Edwards-method, we have determined the roughness exponent $\zeta = 3/4$ and the energy fluctuation exponent $\omega = 1/2$, which fulfill the scaling relation $\omega = 2\zeta - 1$. These

findings have been confirmed by integrating numerically the Burger's equation. Since $\zeta < 1$, there will be no spontaneous condensation of vortices for weak disorder and hence we conclude that the ordered phase is stable.

A further interesting application of our result is that on the XY model in a random field.^{25,26} From the results of Villain and Fernandez²⁷ for this model *without* vortices one finds $\sigma(t) \approx \sigma^* t^{d-2}$, where $\sigma^* = O((4-d)/(d-2))$. Using this t dependence of σ in the Flory argument one concludes $\zeta = 1$ for $2 < d < 4$, apart from a logarithmic correction. From this one should expect that one is in a marginal situation, which deserves further investigation.²⁸

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