Numerical study of the strongly screened vortex-glass model in an external field

Frank Oliver Pfeiffer
Institut für Theoretische Physik, Universität zu Köln, D-50937 Köln, Germany

Heiko Reiger
Institut für Theoretische Physik, Universität zu Köln, D-50937 Köln, Germany and NIC c/o Forschungszentrum Jülich, D-52425 Jülich, Germany

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The vortex-glass model for a disordered high-T_c superconductor in an external magnetic field is studied in the strong screening limit. With exact ground state (i.e., $T=0$) calculations we show that (1) the ground state of the vortex configuration varies *drastically* with infinitesimal variations of the strength of the external field; (2) the minimum energy of global excitation loops of length scale L do *not* depend on the strength of the external field; however, (3) the excitation loops themself depend sensibly on the field. From (2) we infer the absence of a true superconducting state at any finite-temperature independent of the external field. [S0163-1829(99)08133-3]

The gauge or vortex-glass model has become a paradigm in studying amorphous high-T_c superconductors or random Josephson-junction arrays.[1] One essential feature of this model is the possible appearance of a glassy state at low enough temperatures, without which true superconductivity would cease to exist in these disordered materials.[2] From the theoretical side it is now commonly believed that in the absence of screening a true superconducting vortex-glass phase occurs at low enough temperatures.[4–6] If screening is present the original, unscreened $1/r$ interaction of the vortex lines is exponentially shielded beyond a particular length scale λ and the situation seems to change, in particular in the limit in which the screening length is zero (i.e., where vortex lines interact only on-site) the low-temperature vortex-glass phase seems to be destroyed.[7–9,6] In a typical experimental situation[3] the amorphous high-T_c superconductor is put into a homogeneous magnetic field pointing, say, in the z direction. Due to bulk disorder, i.e., inhomogeneities (vacancies, defects, etc.), in the bulk of the sample the vector potential acting on the superconducting phase variables attains a random component (most plausible for granular superconductors[10]), however, still there should be a homogeneous background field superposed on the random part.

Therefore, in this paper we study the question of how is the latter scenario, i.e., the absence of a true superconducting phase in the strongly screened three-dimensional (3D) gauge or vortex-glass model influenced by the presence of a homogeneous external field in one particular space direction. This is done via the investigation of exact ground states of the vortex-glass Hamiltonian and its low-energy excitation. First we analyze the sensibility of the minimum energy configuration with respect to the addition of a homogeneous external field, then we study the low-energy excitations of length scale L in the spirit of the usual domain-wall renormalization-group (DWRG) calculations.[5,6,11]

The lattice model describing the phase fluctuations in a strongly disordered superconductor close to a normal-to-superconductor phase transition is the gauge glass model[1,7]

$$H=-J\sum_{\langle ij\rangle}\cos(\phi_i-\phi_j-A_{ij}-\lambda^{-1}a_{ij})+\frac{1}{2}\sum_{\square}(\nabla\times\mathbf{a})^2, \tag{1}$$

where the first sum runs over all nearest-neighbor pairs $\langle ij\rangle$ on a L^3 simple cubic lattice and the second over every elementary plaquette $\square$, respectively, the phase variable $\phi_i \in [0,2\pi[$ and a_{ij} the fluctuating vector potentials and λ the screening length. $A_{ij}=A_{ij}^{\mathrm{rand}}+A_{ij}^{\mathrm{hom}}$ are the quenched vector potentials consisting of a random component $A_{ij}^{\mathrm{rand}}\in[0,2\pi[$ and a homogeneous component A_{ij}^{hom} modeling an external magnetic field in the z direction. The parameter λ is the bare screening length. A similar Hamiltonian occurs for ceramic (granular) superconductors including the self-inductance of vortex loops.[12,13] For simplicity we set $J=1$. The magnetic field $\mathbf{b_i}$ can be constructed from the quenched vector potentials A_{ij} by a lattice curl, in our case with the homogeneous external field we have

$$\mathbf{b}_i=\frac{1}{2\pi}[\nabla\times\mathbf{A}^{\mathrm{rand}}]_i+\mathbf{B}_i^{\mathrm{ext}}. \tag{2}$$

Obviously the random part fulfills the divergence free condition. We specify the boundary conditions (b.c.) for the vortex-glass Hamiltonian to be periodic in all space directions (corresponding to fluctuating b.c. in the phase variables of the original gauge-glass Hamiltonian[14,15]). Now we choose $\mathbf{B}_i^{\mathrm{ext}}=B\,\mathbf{e}_z$, i.e., the external field points in the z direction and is also divergenceless due to the periodic b.c.

In the *pure* case ($\mathbf{A}^{\mathrm{rand}}=0$) the field strength B simply plays the role of the usual filling factor f counting the number of flux units per plaquette giving rise to the uniformly frustrated XY model (see, e.g., (Refs. 15–17) and references therein) in the unscreened case ($\lambda=\infty$). Here, due to the long-range interaction $\sim 1/r$, the ground state is indeed nontrivial for irrational filling factors. In the continuum limit the flux lines would actually form a hexagonal lattice, the well-known Abrikosov flux-line lattice. For the disordered case one has an interesting interplay between two sorts of frustration: one is also present in the pure case and coming from the external field and the other comes from the quenched disorder. To our knowledge, this problem has not been investigated systematically so far. Here, as a first step, we confine ourselves to the strongly screened case ($\lambda\to 0$), for which the vortex Hamiltonian of Eq. (1) simplifies[8] to

$$H_V^{\lambda\to 0}=\sum_i (\mathbf{J}_i-\mathbf{b}_i)^2. \quad (3)$$

The problem of finding the ground state is actually a minimum-cost-flow problem $\min_{\{\mathbf{J}_i\}}\Sigma_i c_i(\mathbf{J}_i)$ subject to the constraint $(\nabla\cdot\mathbf{J})_i=0$, where $c_i(\mathbf{J}_i):=(\mathbf{J}_i-\mathbf{b}_i)^2$ are the so-called (convex) cost functions. This problem can be solved exactly in polynomial time via combinatorial optimization techniques, as described in Refs. 18, 9.

First we study the sensibility of the ground state with respect to small changes in the external field B. To this end we compare the ground-state configurations of samples with the same quenched disorder and slightly different external field B. Denoting with $\mathbf{J_i}$ the zero-field ($B=0$) ground state and with $\mathbf{J_i}(B)$ the ground state of the same sample in nonvanishing external field B we define the Hamming distance of the two configuration $\mathbf{J}_i$ and $\mathbf{J}_i(B)$ by

$$D_B(L)=\sum_i (\mathbf{J}_i(B)-\mathbf{J}_i)^2, \quad (4)$$

so that a small value of $D_B(L)$ means a strong correlation of the ground states. In Ref. 9 it has been found that an infinitesimal *random* perturbation of the vector potential $\mathbf{A}_{ij}^{\mathrm{rand}}$ leads to a chaotic rearrangement of the ground-state configuration. There it was demonstrated that, like in spin glasses,[19] beyond a particular length scale, the so-called overlap length, the two ground states (perturbed and unperturbed) decorrelate. For the *nonrandom* magnetic-field perturbation we study here we take over this concept and demonstrate the existence of an overlap length l^* scaling with the strength of the external field B like $l^*\propto B^{1/\zeta}$, where ζ is the chaos exponent. For this length scale to exist the finite-size scaling form $D_B(L)=d(L/l^*)=d(L/B^{1/\zeta})$ should hold, which is indeed satisfied, as is shown in Fig. 1. We obtain a relatively large chaos exponent $\zeta=3.8\pm 0.2$. Remarkably this exponent coincides (within the error bars) with the chaos exponent for a *random* perturbation which has been reported to be $\zeta^{\mathrm{rand}}=3.9\pm 0.2$.[9]

In this section we study the scaling behavior of low-energy excitations $\Delta E(L)$ of length scale L (to be defined below) in the presence of an external field, which provides the essential evidence about the stability of the ground state with respect to thermal fluctuations. If $\Delta E(L)$ decreases with increasing length L it implies that it costs less energy to turn over larger domains thus indicating the absence of a true ordered (glass) state at any $T\neq 0$. Usually one studies such an excitation of length scale L by manipulating the b.c. for the phase variables of the original Hamiltonian (1), see Refs. 7,6. One induces a so-called domain wall of length scale L into the system by changing the b.c. of a particular sample from periodic to antiperiodic (or vice versa) in one space direction and measures the energy of such an excitation by comparing the energy of these two ground-state configurations. This is the common procedure for a DWRG analysis, which, however, bears some technical complications[7] and some conceptual ambiguities[5,6] in it.

Here we follow the basic idea of DWRG, we will, however, avoid the complications and the ambiguities that appear by manipulating the b.c. and try to induce the low energy excitation in a different way, as it has first been done by one of us in[9] for the zero-field case. First we clarify what a low-energy excitation of length scale L is in the model under consideration here it is certainly a global vortex loop encircling the 3D torus (i.e., the L^3 lattice with periodic b.c.) once (or several times) with minimum energy cost. How can we induce the above-mentioned global vortex loop, if not by manipulating the b.c.? Schematically the solution is the following numerical procedure: (1) Calculate the exact ground-

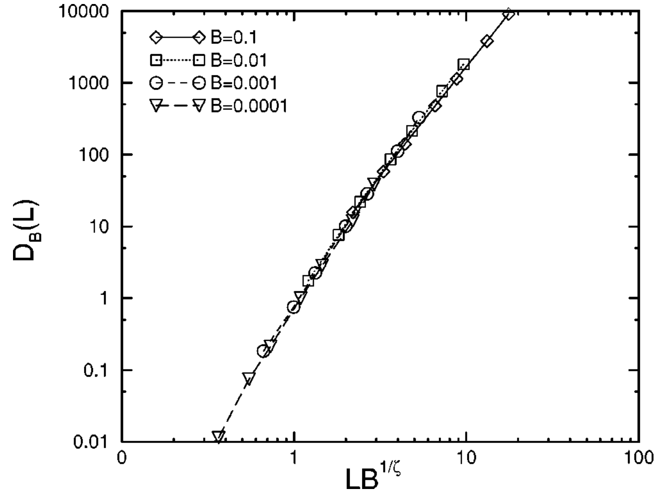

FIG. 1. Scaling plot of the Hamming distance $D_B(L)$ vs $LB^{1/\zeta}$ for $L\leq 32$: 5000 samples for $L\leq 16$, 2000 for $L=24$ and 500 for $L=32$. The chosen values for B are $B=0.0001$, 0.0010, 0.0100, and 0.1000. The best data collapse is achieved by a chaos exponent $\zeta=3.8\pm 0.2$. The error bars are less than the size of the symbols and thus omitted.

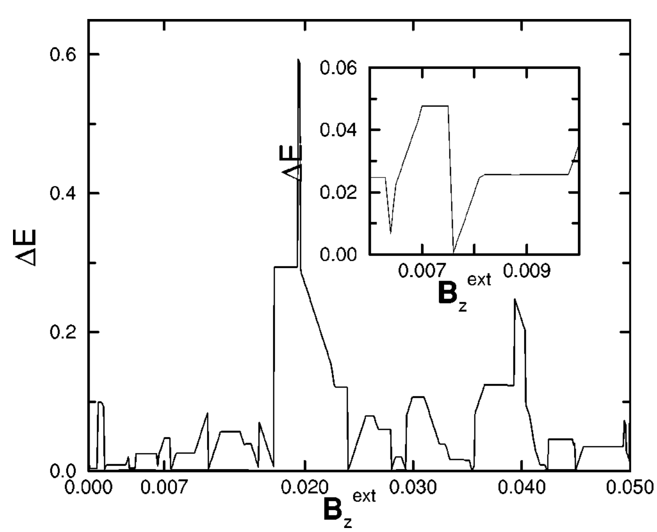

FIG. 2. Defect energy ΔE vs applied magnetic field B for one particular disorder configuration. The field varies between 0 and 0.05 times one flux unit and the system size is $L=24$. The inset enlarges the region that is studied in Fig. 3 in more detail.

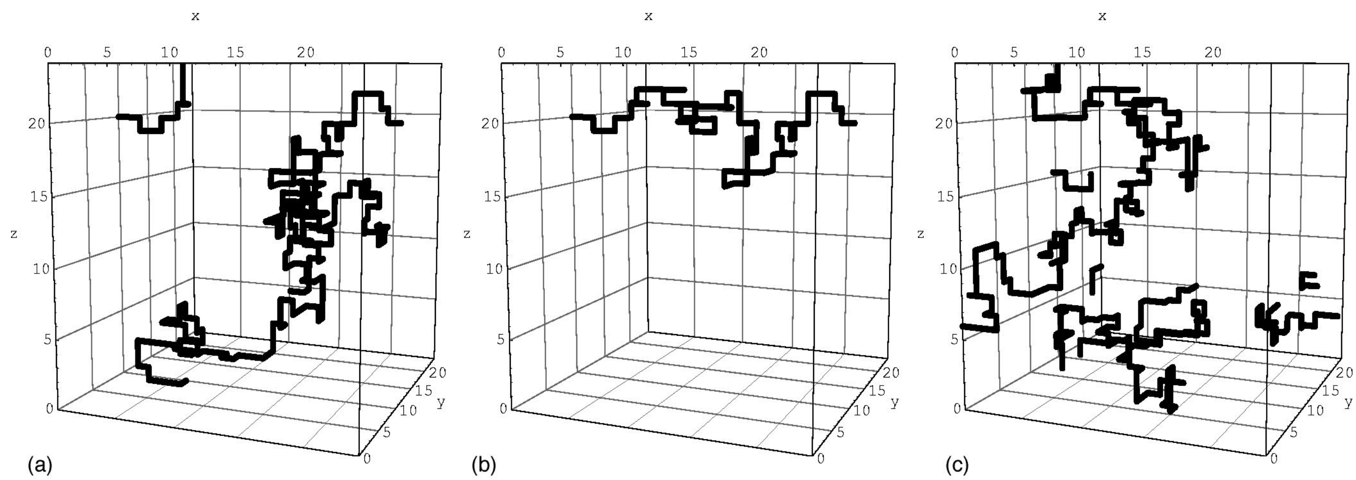

FIG. 3. The minimum energy global excitation loop *perpendicular* to the external field in the z direction is shown for one particular sample ($L=24$) and three different field strengths B (note the periodic b.c. in all space directions). (a) (left) $B\in[0.0065,0.0069]$ is in a range, where the defect energy ΔE varies linear with respect to the field (see inset of Fig. 2). Note that the loop has also winding number $n_z=1$ in the direction *parallel* to the external field. Hence, $\partial\Delta E/\partial B=2L$. (b) (middle) The same sample as in (a) with $B\in[0.0070,0.0075]$. In this interval the defect energy is constant, no loop along the direction of the applied field occurs. (c) (right) The same sample as in (a) and (b) with $B\in[0.0076,0.0081]$. The system is very sensible to the variation of applied field ΔB. Even for a small change by $\Delta B=0.0001$ the form of the excitation loop changes drastically.

state configuration $\{\mathbf{J}^0\}$ of the vortex Hamiltonian (3); (2) Determine the resulting global flux along, say, the x axis $f_x=(1/L)\Sigma_i J_i^{0x}$; (3) Study a minimum-cost-flow problem in the actual cost for increasing the flow in the x direction $\Delta c_i^x=c_i(J_i^{0x}+1)-c_i(J_i^{0x})$ is smoothly modified letting the cost of a topologically simple connected loop unchanged and only affecting global loops. (4) Reduce the Δc_i^x until the optimal flow configuration $\{\mathbf{J}^1\}$ for this min-cost-flow problem has the global flux (f_x+1), corresponding to the so-called *elementary* low-energy excitation on the length scale L; (5) Finally, the defect energy is $\Delta E=H(\{\mathbf{J}^1\})-H(\{\mathbf{J}^0\})$.

Two remarks (1) In the pure case this procedure would not work, since at some point spontaneously *all* links in the z direction would increase their flow value by one. It is only for the disordered case with a continuous distribution for the random variables $\mathbf{b}_i$ that a unique loop can be expected. (2) In the presence of a homogeneous external field one has to discriminate between different excitation loops: those parallel and those perpendicular to the external field need not to have the same energy.

As for the zero-field case[9] one expects for the disorder-averaged excitation energy (or defect energy)

$$[\Delta E(B,L)]_{\mathrm{av}}\sim L^{\theta}, \tag{5}$$

where B is fixed, $[\cdots]_{\mathrm{av}}$ denotes the disorder average and θ is the stiffness exponent and its sign determines whether there is a finite-temperature phase transition or not, as explained above. If $\theta<0$, i.e., the transition to a true superconducting vortex state appears only at $T=0$.[8,9,19]

For a single configuration we find that a change of the external magnetic field B drastically affects the defect energy ΔE (Fig. 2). ΔE is a piecewise linear function that behaves in particular intervals $[B_a,B_b]$ as

$$\partial\Delta E/\partial B=2\cdot L\cdot n_z, \tag{6}$$

which can be understood as follows: the external field varies continuously and the interger valued flow changes only in discrete steps, thus the minimum energy excitation loop may not change in a whole interval say $[B_a,B_b]$. In this interval ΔE changes linearly with B since $H(\mathbf{J}^1)-H(\mathbf{J}^0)$ is simply proportional to length of the excitation loop in the z direction, which is $n_z\cdot L$, with n_z the integer winding number of the loop in the z direction. Furthermore, the excitation loops themselves change their form dramatically, as it is exemplified in Fig. 3. Only small parts of the loop seem to persist over a significant range of the field strength, see for instance in the vicinity of the plane $z=20$ in Fig. 3. Now we study the behavior of the disorder-averaged energy of excitation loops *perpendicular* to the applied magnetic field along, say, the z

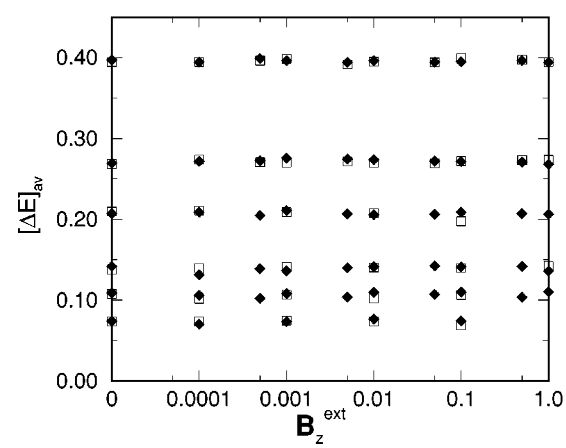

FIG. 4. Linear plot of the external magnetic field vs the defect energy $[\Delta E]_{av}$ for $L=4,6,8,12,16,24$ (top to bottom). The elementary excitation parallel □ and perpendicular full diamond to the external magnetic field $\mathbf{B}_z^{\mathrm{ext}}$. For each plotted point the number of samples varied between 20 000 for the smallest sizes to 1000 for the largest sizes. The error bars of the excitations are less than the size of the symbols and thus omitted.

direction (full diamonds in Fig. 4). Note that it is only necessary to study the situation $B \in [0,1]$, since all physical properties of the vortex glass Hamiltonian (3) are periodic in the strength of the external field B, i.e., the filling factor. As can easily be seen in Fig. 4 the defect energy $[\Delta E(B,L)]_{\mathrm{av}}$ is independent of the value of B. For any fixed value of B the finite-size scaling relation (5) is confirmed and gives $\theta = -0.95 \pm 0.04$; cf. Ref. 9. This behavior of excitations perpendicular to the applied field depends neither on the length of the system in the z direction nor on the topology in this direction: we also studied the situation for elementary excitation loops (open boxes in Fig. 4) *parallel* to the external field and in which the vortex Hamiltonian (3) lives on a lattice with free instead of periodic b.c. in the z direction. In the latter case the external field has an appropriate source and sink outside the system. For all cases we find here the same result: $[\Delta E(B,L)]_{\mathrm{av}}$ is independent of the external field B.

We have studied the 3D vortex-glass model in the strong screening limit in the presence of a homogeneous external field. The ground state is extremely sensible to small external field variations and its configurations at different field values B and $B + \Delta B$ decorrelate beyond the overlap correlation length $l^* \sim \Delta B^{1/\zeta}$ with a chaos exponent $\zeta = 3.8 \pm 0.2$ This value agrees within the error bars with the chaos exponent for random perturbations of the quenched disorder.[9] For individual disorder configurations the change of the defect energy $\Delta E_L(B)$ with respect to the applied field B is piecewise linear, analytic, and accompanied by a drastic deformation of the minimum energy global excitation loop. On the other hand, the disorder-averaged value of the defect energy $[\Delta E_L(B)]_{\mathrm{av}}$ is independent of the external field B in strength and direction and thus the same for the scaling behavior, i.e., identical to the case $B = 0$ in Ref. 9. Therefore, as in the $B = 0$ case, we infer the absence of a true superconducting low-temperature phase.

Concluding, we would like to note that it would be interesting to perform the same analysis for nonvanishing screening length and for the unscreened case, where due to the long-range repulsion of the vortex lines important new physics might appear, in particular for a homogeneous external field.

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