Chaos in the random field Ising model

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The sensitivity of the random field Ising model to small random perturbations of the quenched disorder is studied via exact ground states obtained with a maximum-flow algorithm. In one and two space dimensions we find a mild form of chaos, meaning that the overlap of the old, unperturbed ground state and the new one is smaller than 1, but extensive. In three dimensions the rearrangements are marginal (concentrated in the well defined domain walls). Implications for finite temperature variations and experiments are discussed. [S1063-651X(98)06710-5]

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The concept of *chaos* in disordered systems refers to the sensitivity of their equilibrium state (at finite temperatures) or ground state (at zero temperature) with respect to infinitesimal perturbations. In spin glasses [1], for instance, it is well known that small changes of parameters like temperature or external field cause a complete rearrangement of the equilibrium configuration [2,3]. This has experimentally observable consequences like reinitialization of aging in temperature cycling experiments [4], and has also been investigated in numerous theoretical works [5].

A slight random variation of the quenched disorder has the very same effect on the ground state configurations. Although of similar origin, chaos with respect to temperature changes is harder to observe than chaos with respect to disorder changes [6], and the latter phenomenon has been used to quantify spin glass chaos in numerical investigations [3,7].

This type of chaos was actually later discovered in another, simpler random system, the directed polymer in a random medium [8–10], which is equivalent to a domain wall in a random bond ferromagnet. The interface displacement as a reaction to infinitesimal random changes of bond strengths obeys particular scaling laws with exponents related to the well-known interface roughness exponent χ [9,10].

In this paper we consider the random field Ising model [1] and study, for the first time to our knowledge, the sensitivity of its ground state with respect to small changes in the random field configurations. It turns out that the emerging picture is very reminiscent of chaos in spin glasses and random interfaces. This statement is quantified by the following phenomenological picture [3,9].

Consider a random Ising system defined, for instance, by the Hamiltonian

$$H = -\sum_{\langle ij\rangle} J_{ij} S_i S_j - \sum_i h_i S_i, \qquad (1)$$

where $S_i = \pm 1$ are Ising spins, $\langle ij\rangle$ indicates nearest neighbor pairs on a D-dimensional lattice of, say, linear size L, and J_{ij} denote interaction strengths and h_i local fields, both quenched random variables obeying some distribution (continuous, in order to exclude ground state degeneracies). The case when is J_{ij} Gaussian (with mean zero and variance 1) and $h_i = 0$ is the *spin glass* (SG) model. The case when $J_{ij} \geqslant 0$, and $h_i = 0$ is the *random bond ferromagnet* model (RBFM). The case when $J_{ij} = J$, and h_i is Gaussian (with mean zero and variance h_r) is the *random field Ising model* (RFIM). In order to study the sensitivity of the ground state of these systems with respect to small changes in the quenched disorder, we can apply a random perturbation of amplitude $\delta \ll 1$ to any of the quenched random variables. As a consequence the *new* ground state will differ from the old one.

The RFIM ground state changes when the domain structure changes (for purely ferromagnetic states this argument does not work). One can estimate when the two ground states will be uncorrelated, beyond a length scale L^*. This can be found considering domain walls with an Imry-Ma [11] type argument [3,9]: The energy $E_{\rm flip}$ to flip droplets or domains or excitations of size L scales like L^θ, where θ is the energy fluctuation exponent (θ is denoted y in the SG context [3]; it does *not* stand for the violation of hyperscaling exponent at the critical point of the RFIM [12]). The energy change due to the random perturbation $E_{\rm rand}$ scales like $\delta L^{d/2}$, where $d = d_s$ is the fractal dimension of the droplet's surface in the SG case, $d = D-1$ is the interface dimension in the RBIM, and $d = D$ in the RFIM case. The decorrelation takes place when $E_{\rm rand}(L) > E_{\rm flip}(L)$, i.e., for $L > L^* \sim \delta^{-1/\lambda}$, with $\lambda = d/2 - \theta$. In SG jargon L^* is called the overlap length, and λ is denoted ζ, the chaos exponent [3,7].

Two remarks are in order: first, as already pointed out in Refs. [9] and [10] for $L < L^*$, the ground state is slightly altered by the random perturbation. This is, however, an effect of the interplay between elastic energy and $E_{\rm rand}$. This leads to displacements of the domain wall of size $\Delta x \sim \delta L^\alpha$ with $\alpha = \lambda + \chi$, where χ is the roughness exponent. The roughness exponent [13] and the energy fluctuation exponent θ are related via $\theta = 2\chi + D - 3$ [9].

The second remark concerns the RFIM case: for $D \leqslant 2$ the concept of a macroscopic domain wall fails, and the above considerations can only be transferred *cum grano salis*. This means that they are sensible only for $L \ll \xi \sim \exp(-h_r^2/A)$, the typical size of domains in the two-dimensional (2D) RFIM [14,15].

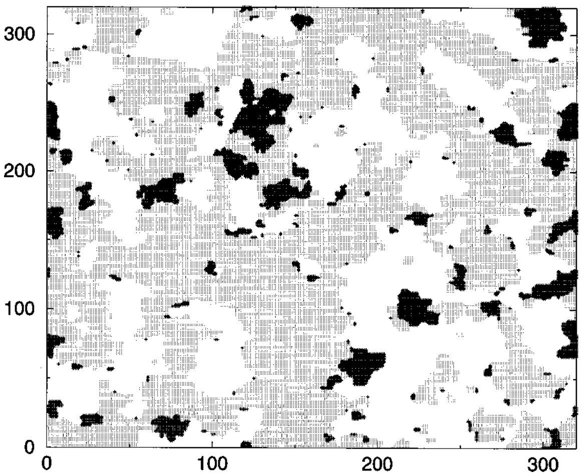

FIG. 1. A ground state plus the perturbation-induced changes. The original spin orientations are indicated in grey for $S_i=+1$ and white for $S_i=-1$. The flipped spins are indicated in black. $L=320$, $\Delta=2$, and $\delta=0.1$ (see text).

In three dimensions the situation is different: The concept of a domain wall is well defined, and we obtain, with the estimate for the roughness exponent $\chi=\frac{2}{3}$ [13] and, consequently, $\theta=\frac{4}{3}$, the result $\lambda=\frac{1}{6}$, i.e. $L^*\sim\delta^{-1/6}$. The typical displacement of a domain wall thus is, as above, $\Delta x\sim L^{\alpha}$, with $\alpha=\frac{2}{3}$ for $L\ll L^*$ and $\alpha=\frac{5}{6}$ for $L\gg L^*$.

If we take the above arguments to be serious for the RFIM in two dimensions, the overlap length L^* turns out to be formally infinite (since with $\theta=1$ one has $\lambda=0$), where one of course has to be careful due to logarithmic corrections to the energy $E_{\rm flip}$. Thus the mechanism by which rearrangements take place is due to the interplay between elastic energy and $E_{\rm rand}$. Moreover, in the 2D RFIM, the typical displacement of domain walls should scale as $\Delta x\sim\delta L$ for $L\ll\xi$, since $\alpha=\lambda+\chi=1$. As a consequence the correlation or overlap between the old unperturbed ground state S_i and the new one $S_i'(\delta)$,

$$q=\frac{1}{L^D}\sum_i S_i S_i'(\delta), \tag{2}$$

behaves like $1-q\sim L^{\alpha-1}$, and therefore q should be of order $O(1)$, depending on the probability with which domain wall displacements occur.

In what follows we present results of exact ground state calculations for 1D spin chains and for 2D systems. We use a random field distribution and a perturbation distribution that have a constant probability density between $-\Delta$ and Δ and $-\delta/2$ and $\delta/2$, respectively, and set $J_{ij}=1$. Figure 1 shows an example of a large 2D ground state ($L=320$) with the two spin orientations shown in white and gray, respectively, and the *flipped* spins in black. There are two features one should note. First, the size of the system is larger than the critical length scale needed for ground state breakup, and the magnetization is practically zero. Second, the flipped spins form a number of clusters of varying size, that seem to concentrate on the *cluster boundaries* of the original ground state.

Figure 2 shows what happens as one sweeps the RF strength (Δ). In arbitrary dimensions, the limit $\Delta\to\infty$ goes over to a site percolation problem, i.e., the local RF orientation gives the spin state at a site. In that limit the overlap q is determined by the probability of the applied perturbation δ to change the orientation. For somewhat smaller fields q becomes smaller, in an apparently linear fashion, as Δ changes. In the 1D case the overlap is not sensitive to the system size above a certain threshold in Δ, below which the overlap quickly increases to unity again, which indicates a typical domain size. The overlap seems to become a δ-dependent constant in the thermodynamic limit and for $\Delta\to 0$.

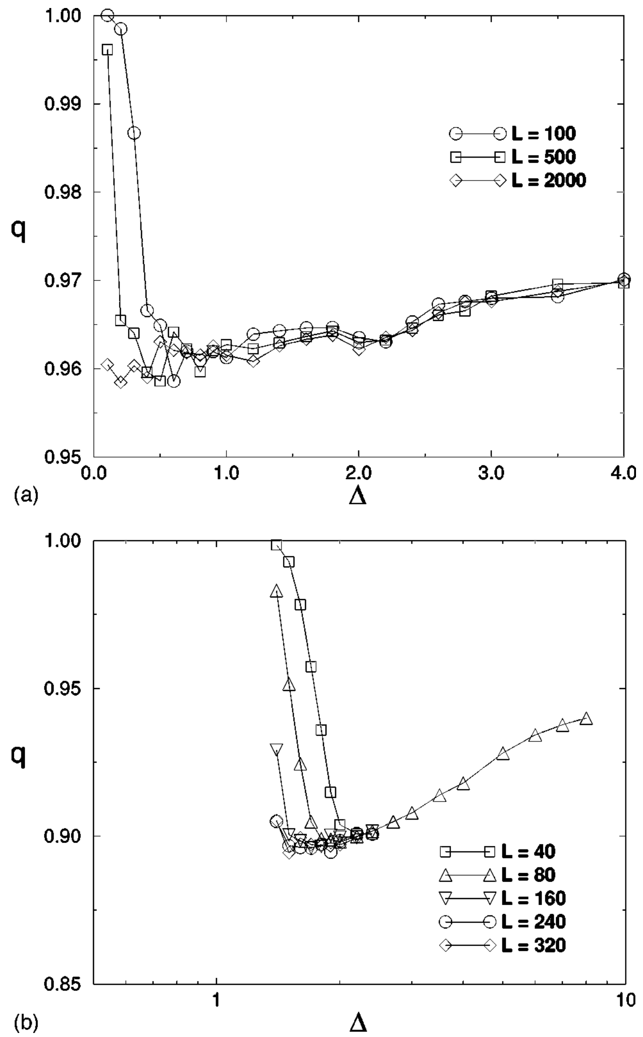

FIG. 2. (a) (Top) Scaling of the overlap parameter with random field strength for the 1D spin chain. (b) (Bottom) Scaling of the overlap in two dimensions for the system sizes $L=40$, 80, 160, 240, and 320, and for $\delta=0.1$.

This 1D behavior can be understood as follows. For simplicity let us assume that the first spin is fixed to be up, i.e., $S_0=+1$. Then the total random field energy at site n is given by $H_r=\Sigma_{i=1}^n h_i$ in the unperturbed system, and $H_n'=H_n+\Delta_n$ with $\Delta_n=\Sigma_{i=1}^n\delta_i$, in the perturbed one. If h_i and δ_i are independently distributed variables with zero mean and variance $h_r=[h_i^2]_{\rm av}$ and $\delta_r=[\delta_i^2]_{\rm av}$, respectively, the variables H_n and Δ_n are (for $n\gg 1$) Gaussian with mean zero and variance nh_r and $n\delta_r$, respectively. The probability distribution $P(H_n,H_n')$ is simply given by

$$P(H_n,H_n')=\int d\Delta_r P(H_n)P(\Delta_n)\delta(H_n+\Delta_n-H_n').$$

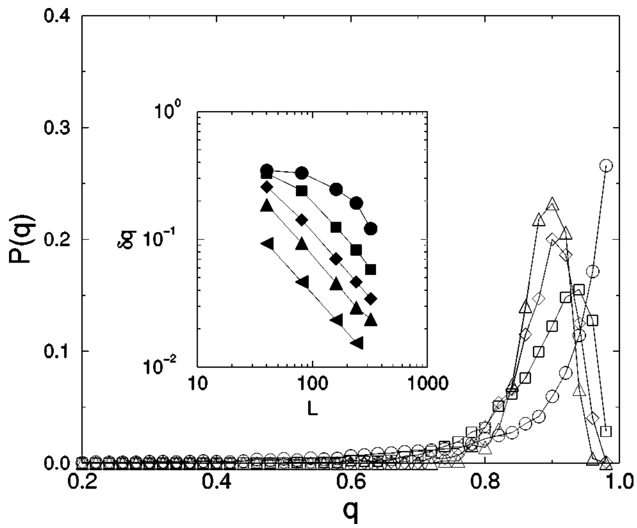

FIG. 3. Probability distributions of the overlap q for $\Delta=1.8$ for the system sizes $L=40,\ldots,320$. The inset shows the standard deviations of the overlap pdf's for $\Delta=1.4$, 1.6, 1.8, 2, and 2.4.

Now the total RF fluctuations H_n and H_n' produce domains if their magnitude is large enough to overcome the ferromagnetic coupling: suppose that $S_i=+1$ and $H_i>-J$ for $i=1,\ldots,n$ (i.e., a plus domain), but $H_{n+1}<-J$; then S_{n+1} will be flipped, i.e., $S_{n+1}=-1$, and a new (minus) domain starts. For large enough typical domain sizes the total RF fluctuations become large: one can neglect J and assume that only the signs of H_n and H_n' determine the ground state (note that this is different from the high field region $h_n \gg J$, in which the local random fields h_i dominate). Thus the probability of S_n and S_n' being equal is given by

$$p(S_n=S_n')=\int dH_n dH_n' P(H_n,H_n')\,\theta(H_n H_n'), \qquad (3)$$

where θ is the step function. A straightforward calculation yields $p(S_n=S_n')=1-(1/\pi)\delta_r/h_r+O(\delta^2)$. For the data shown in Fig. 1(a), in which $h_r^2=\Delta^2/3$ and $\delta_r^2=\delta^2\Delta^2/12$ with $\delta=0.1$, we have $\delta_r/h_r=0.05$ and hence $q=-1+2p(S_r=S_r')\approx 0.97$, agreeing roughly with the numerical results for $h_r\to 0$ in the limit $L\to\infty$.

The 2D behavior is depicted in Fig. 2 for $\delta=0.1$. The number of simulations is 10 000 for $L=40$ and 80, 4000 for $L=160$, 1000 for $L=240$, and 500 for $L=320$. The generic behavior of the overlap is as for the 1D chain: $q(\Delta)$ is roughly linear until the regime of small fields ($\Delta\leq 2$), after which it seems to saturate to a δ-dependent value $q(\delta)$. The crossovers (increase of q with decreasing Δ) are due to the ground state breakup mechanism. For small systems the ground state is ferromagnetic, except for a limited number of domains of the opposite spin orientation. The decrease in q is caused by the effect of the ground state becoming more and more uniform (magnetization $|m|\to 1$). Otherwise the behavior strongly resembles the 1D case.

The thermodynamic behavior of the overlap is also visible in the statistics of overlap distributions. Figure 3 shows how the probability distribution $P(q)$ of q behaves with varying system size and for $\Delta=1.8$ (the data are the same as presented in Fig. 2). For all systems $P(q)$ is peaked at $q=1$, but as L is increased a peak appears in the distribution, resembling a Gaussian. The inset of Fig. 3 shows the standard deviation δq of $P(q)$ for varying Δ as a function of the system size L. Except for the by now standard crossover for small L and Δ, we observe that the width of the distribution *decreases*, which signals that in the thermodynamic limit $P(q)$ approaches a δ-function-like sharp distribution. The crossover exponent c, defined with $\delta q\sim L^{-c}$, seems to be exactly 1 ($c=1$).

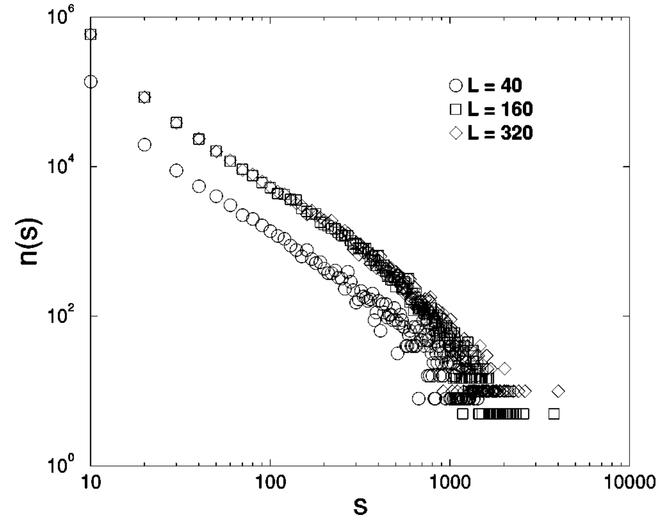

FIG. 4. Cluster size distributions of clusters of flipped spins for $\delta=0.1$, $L=40$, 160, and 320, and $\Delta=2$.

The mechanism by which q is determined is illustrated in Fig. 4. The size distribution of flipped clusters $n(s)$ converges with L to a power law, $n\sim s^{-1.6}$, with a cutoff that depends very weakly if at all on L. This has to be so for the overlap not to diverge to zero in the thermodynamic limit, since one can write $1-q$ as an integral over $n(s)$: an L-dependent cutoff would imply that q would decrease continuously.

Finally, in Fig. 5, we demonstrate that $1-q\sim\delta$ for small δ. This follows from the scaling arguments presented for 2D RFIM domain walls and the 1D RF chain.

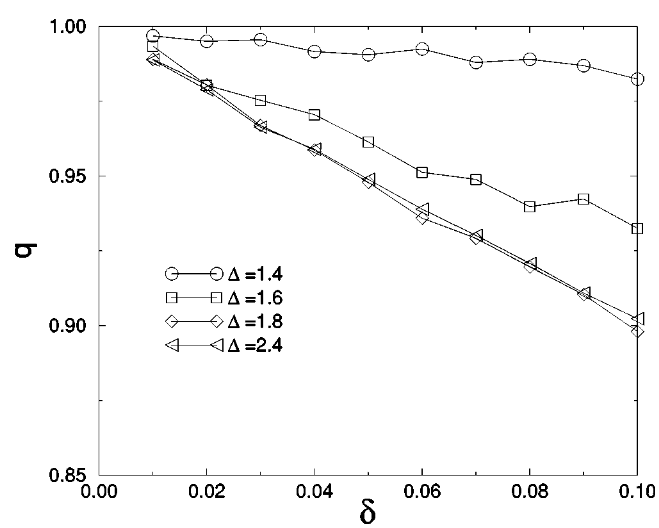

FIG. 5. Dependence of the overlap q on the perturbation strength δ for weak and strong magnitudes (Δ), $L=80$.

In this paper we have considered the stability of the random field Ising model to small perturbations. Unlike in spin glasses, it turns out that the RFIM ground state shows a weak form of chaos, similar to directed polymers or random bond Ising model domain walls. The overlap q attains its value from fluctuations of the domain walls, in both one and two dimensions. Thus the ground state stays almost intact. The ground state domains are robust against external perturbations since, most likely, the field excess of a domain is extensive $(\Sigma h_i \sim V)$. For the RFIM in three dimensions, the prediction of the domain wall scaling argument is that q should converge to unity since the domain wall displacement exponent α here is $\frac{5}{6}$: the displacement of a domain wall on large enough length scales is $\Delta x \sim L^{\alpha}$, and therefore $1-q \propto L^{\alpha-1} \rightarrow 0$. Moreover, in both limits $h_r/J \rightarrow 0$ and $h_r/J \rightarrow \infty$; i.e., deep in the ferromagnetic phase and deep in the paramagnetic phase, it is trivial that $q \rightarrow 1$.

One would like to extend the argumentation to changes in temperature, as is common for spin glasses and random-bond-type directed polymers. In spin glasses chaos is inti mately linked to the nonequilibrium correlation length, which gives rise to measurable consequences in, e.g., temperature cycling experiments that measure the out-of-phase susceptibility. Here, however, repeating the scaling argument of the domain wall for temperature changes results in a displacement exponent which does not produce any extensive changes in the overlap. In two dimensions the predicted outcome is simply that of a random walk $(\Delta x \sim L^{1/2})$. In other words, assuming that typical valleys in the energy landscape are separated by an energy given by the energy fluctuation exponent gives completely different results for temperature and ground state chaos than for random bond disorder. This discussion is intimately related to coarsening and aging in the RFIM; one should note that so far, to our knowledge, there have been no simulation results that address these questions directly.

Note added in proof. Recently we became aware of temperature cycling experiments in a random field system [16], where indications for the (partial) reinitialization of aging have been reported. These are probably not caused by chaotic rearrangements of domain walls, but originate from the existence of slow and fast domains (see [16]).

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