PHYSICAL REVIEW E **75**, 030104(R) (2007)

Superaging in two-dimensional random ferromagnets

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(Received 13 November 2006; published 30 March 2007)

We study the aging properties, in particular the two-time autocorrelations, of the two-dimensional randomly diluted Ising ferromagnet below the critical temperature via Monte Carlo simulations. We find that the autocorrelation function displays additive aging $C(t,t_w)=C_{st}(t)+C_{ag}(t,t_w)$, where the stationary part C_{st} decays algebraically. The aging part shows anomalous scaling $C_{ag}(t,t_w)=\mathcal{C}[h(t)/h(t_w)]$, where $h(u)$ is a nonhomogeneous function excluding a t/t_w scaling.

DOI: 10.1103/PhysRevE.75.030104 PACS number(s): 05.40.−a, 75.10.Nr, 64.75.+g

The phase ordering kinetics in pure systems has attracted much attention in the last years [1]. A common scenario, for instance, for ferromagnets after a fast quench from above to below the ordering temperature is a continuous domain growth governed by a single length scale that depends algebraically on the time t_w after the quench. The existence of this length scale quite frequently determines also the scaling properties of other dynamical nonequilibrium quantities like the two-time autocorrelation function $C(t,t_w)$, which describes the correlations between the spin configurations at the time t_w after the quench and a later configuration at a time $t+t_w$. It gives rise to what is called *simple aging* in the context of glassy systems [2]: $C(t,t_w)$ depends for large times t and t_w only on the scaling variable t/t_w. This behavior is rather well established by analytical works in various nonrandom models [3], and it has been corroborated by a large amount of numerical work [2].

Much less analytical results are available for disordered ferromagnets, where numerical simulations thus play an important role. A recent numerical study of the relaxational dynamics in two-dimensional random magnets [4] found evidence for a power-law growth $L(t)\propto t^{1/z}$ of the aforementioned length scale $L(t)$. The dynamical exponent z turned out to depend both on temperature T and disorder strength and to behave as $z\propto 1/T$ at low temperatures T. The latter is compatible with activated dynamics of pinned domain walls over *logarithmic* free-energy barriers (rather than power law [5]). The apparent existence of a single length scale that grows algebraically was confirmed by a recent numerical work [6], where it was furthermore claimed that the response function is well described by local scale invariance [7]. In spite of this, the correlation function showed systematic deviations from a simple t/t_w scaling [6] (although simple aging seems to work well in $d=3$ [8]). In [6], the autocorrelation was then compared to the scaling form $C(t,t_w)\sim t^{-x}\tilde{c}(t/t_w)$, which usually holds at a critical point with $x>0$ [9]. However, a fit of this form to the numerical data obtained in [6] (and to ours as we will report below) yielded *negative* exponents x, which is unphysical. The aim of this paper is to suggest an alternative picture originally applied in the context of aging experiments in glasses [10] and spin glasses [11].

We study the site diluted Ising model (DIM) defined on two-dimensional square lattice with periodic boundary conditions, and described by the Hamiltonian

$$H=-\sum_{\langle ij\rangle}\rho_i\rho_j s_i s_j, \tag{1}$$

where $s_i=\pm 1$ are Ising spins, the ρ_i's are quenched, identical, and independent random variables distributed according to the probability distribution $P(\rho)=p\delta_{\rho,1}+(1-p)\delta_{\rho,0}$. Above the percolation threshold $p>p_c$, with $p_c\simeq 0.593$ [12], the equilibrium phase diagram is characterized by a critical line $T_c(p)$ [with $T_c(p_c)=0$] which separates a ferromagnetic phase at low temperature T from a paramagnetic one at high T. Here we focus on the relaxational dynamics of this system (1) following a quench in the ferromagnetic phase, $T<T_c(p)$. At the initial time $t=0$, up and down spins are randomly distributed on the occupied sites when it is suddenly quenched below $T_c(p)$ where it evolves according to Glauber dynamics (corresponding to the heat-bath algorithm) with random sequential update, representing a discretized version of model *A* dynamics, i.e., for nonconserved order parameter.

In the following we focus on the two-times $t>0$ autocorrelation function $C(t,t_w)$ which is defined as

$$C(t,t_w)=\frac{1}{L^2}\sum_i \overline{\langle s_i(t+t_w)s_i(t_w)\rangle}, \tag{2}$$

where $\langle\cdots\rangle$ and $\overline{\ldots}$ stand for an average over the thermal noise and the disorder, respectively, and where L is the linear system size. In our simulations, $L=512$ and $C(t,t_w)$ is obtained by averaging over 50 different disorder configurations. In Fig. 1 we show a plot of $C(t,t_w)$ as a function of the time difference t and for different waiting times t_w. These data were obtained for $p=0.75$ and $T=0.7T_c(p=0.75)$.

The data shown in Fig. 1 on a log-log plot suggest a power-law behavior defining the off-equilibrium exponent λ [13]

$$C(t,t_w)\propto t^{-\lambda/z}, \quad t\gg t_w \tag{3}$$

which is, as we can see in Fig. 1 weakly dependent on t_w. We have checked that our simulations reproduce the well-known values for the pure case, with $z_{\text{pure}}=2$ [1] and $\lambda_{\text{pure}}=1.25$ conjectured to be exact in Ref. [14]. Figure 2 shows a plot of λ as a function of $T/T_c(p)$ for $p=0.75$, 0.8, and 0.89.

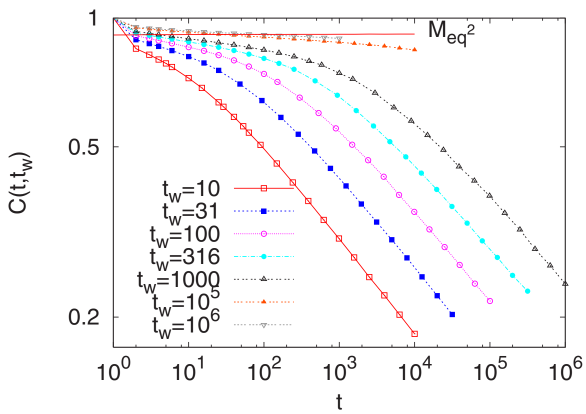

FIG. 1. (Color online) $C(t,t_w)$ plotted in the double logarithmic scale as a function of t for different waiting times t_w=10, 31, 100, 316, 1000, 10^5, and 10^6. $M_{\rm eq}$ is the equilibrium magnetization computed with the Swendsen-Wang algorithm (see below). Data set is obtained for p=0.75 at temperature T=0.7T_c.

As we can see, $\lambda(T,p)$ depends rather weakly on T (in contrast with z) and p. Besides, the obtained values violate the lower bound proposed in Ref. [14], $\lambda \geq d/2$. Such a violation was also obtained analytically for random-field XY model in d=2 [15].

We now focus on the scaling form of $C(t,t_w)$ as a function of both times t, t_w. For nondisordered ferromagnets with purely dissipative dynamics, one expects that $C(t,t_w)$ depends only on the ratio $\ell(t)/\ell(t_w)$, i.e.,

$$C_{\rm pure}(t,t_w) = F_{\rm pure}[\ell(t)/\ell(t_w)] \tag{4}$$

with $\ell(t) \propto t^{1/2}$. This has been corroborated by numerical simulations [2] as well as analytical results in exactly solvable limits [1,3]. As shown by Paul *et al.* [4], a power-law domain growth is also observed for the present disordered system, which suggests to plot, here also, $C(t,t_w)$ as a function of t/t_w: this is depicted in Fig. 3(a).

Here one sees that this scaling form does not allow for a good collapse of the curves for different t_w. The deviation

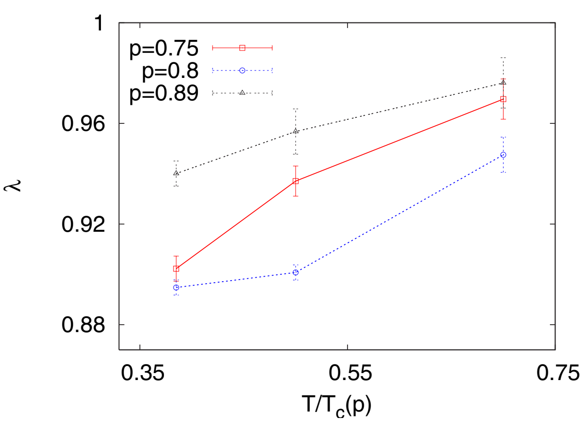

FIG. 2. (Color online) Exponent λ, extracted using Eq. (3), plotted as function of the reduced temperature $T/T_c(p)$. These values, which violate the lower bound $\lambda \geq d/2$ has to be compared with the one for the pure system $\lambda_{\rm pure}$=1.25.

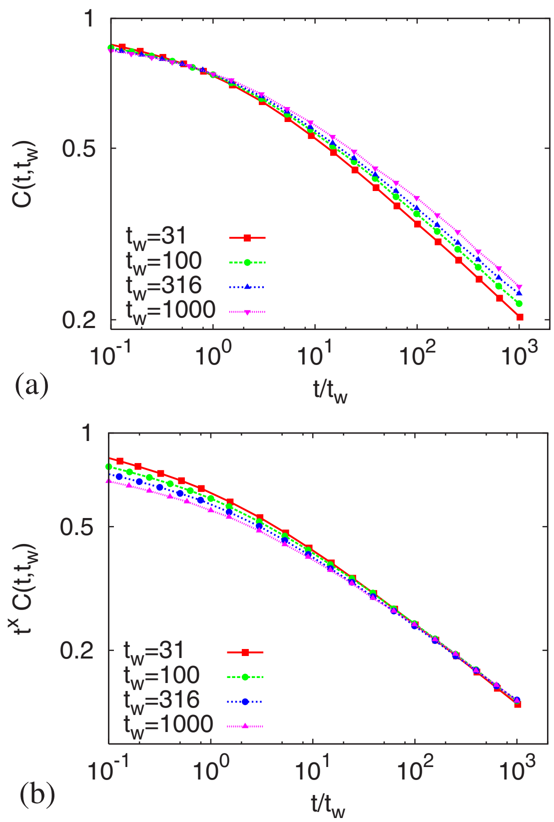

FIG. 3. (Color online) (a) $C(t,t_w)$ as a function of t/t_w for different t_w=31, 100, 316, and 1000. (b) $t^x C(t,t_w)$ as a function of t/t_w. x=−0.04 is obtained from the best data collapse. Data set is obtained for p=0.75 at temperature T=0.7T_c.

from this scaling form is indeed systematic and we have checked that the disagreement with such a scaling persists even for larger waiting time t_w. We have also obtained that simulations for other values of p and $T/T_c(p)$ show the same deviations from t/t_w scaling.

In Ref. [6] the random bond ferromagnet, which is expected to display qualitatively the same behavior as the DIM, was studied and there, the autocorrelation was compared to the scaling form

$$C(t,t_w) = t^{-x(T,p)}\tilde{c}(t/t_w) \tag{5}$$

which works well for critical dynamics [9] as well as some disordered systems such as spin glasses in dimension d=3 [16] or an elastic line in random media [17] with a *positive* exponent $x(T,p)>0$. However, in Ref. [6] a *negative* value for $x(T,p)$ was obtained by fitting the data for $C(t,t_w)$ to Eq. (5). We also get a good data collapse for our data, as shown in Fig. 3(b) when using a negative exponent x. The best collapse according to Eq. (5) is obtained for x=−0.04<0 (for p=0.75 and T=0.7T_c). The fact that $x<0$ would mean that $C(yt_w,t_w)$ grows without bounds when $t_w \to \infty$ (keeping $y \gg 1$ fixed), which is unphysical. This implies that Eq. (5) is not the correct scaling form for $C(t,t_w)$, for which reason we search for an alternative picture. First we point out that as t_w increases $C(t,t_w)$ clearly displays the formation of a plateau (see Fig. 1). This suggests an additive structure, as expected

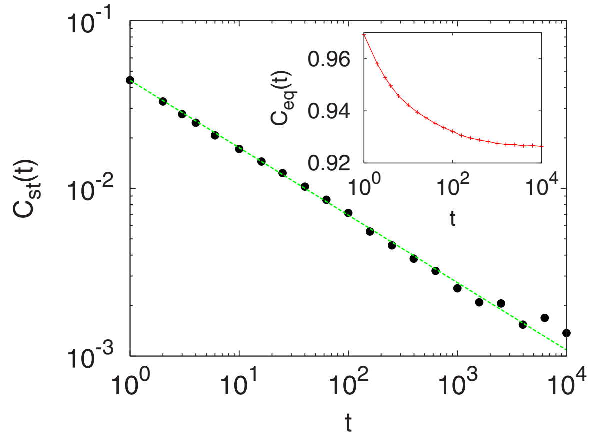

FIG. 4. (Color online) $C_{st}(t)$ plotted in the double logarithmic scale as a function of t. The line corresponds to $\eta=0.4$ in Eq. (7). Inset: $C_{\rm eq}(t)$ as function of t. Data set is obtained for $p=0.75$ and $T=0.7T_c$.

in the ferromagnetic phase [and in contrast to the multiplicative scaling found at $T_c(p)$ in random ferromagnets [18]]:

$$C(t,t_w) = C_{st}(t) + C_{ag}(t,t_w) \tag{6}$$

such that $\lim_{t\to\infty} C_{st}(t)=0$ and $\lim_{t\to\infty}\lim_{t_w\to\infty} C(t,t_w)=M_{eq}^2$ where M_{eq} is the equilibrium magnetization.

We first focus on the stationary component $C_{st}(t)$ in Eq. (6), for which analytical predictions exist relying on droplet models [19]. To study this part numerically, we first equilibrate the system using the Swendsen-Wang algorithm [20] and then let the system evolve according to Glauber dynamics starting with such an equilibrated initial configuration. We denote $C_{\rm eq}(t,t_w)$ the (equilibrium) autocorrelation function (2) computed using this protocol and we have checked that it is indeed independent of t_w, $C_{\rm eq}(t,t_w)\equiv C_{\rm eq}(t)$.

In the inset of Fig. 4 we plot $C_{\rm eq}(t)$ as a function of t for $p=0.75$ and $T=0.7T_c$. In agreement with previous analytical predictions [19], these data can be nicely fitted to $C_{\rm eq}(t)=C_{st}(t)+M_{\rm eq}^2$ with $M_{\rm eq}^2=0.925$ and a power-law behavior,

$$C_{st}(t) \propto A t^{-\eta(T,p)}. \tag{7}$$

This is depicted in Fig. 4, where we show a plot of $C_{st}(t)$ as a function of t for $p=0.75$ and $T=0.7T_c$, for these parameters, one finds $\eta=0.40(2)$.

We now come to the aging part $C_{ag}(t,t_w)$ in Eq. (6), by first noticing that simple aging also does not hold for $C_{ag}(t,t_w)$. Inspired by a picture originally suggested in the context of aging experiments in glasses [10] and spin glasses [11], and also occurring within the analytical solution of the nonequilibrium dynamics mean-field spin glasses [2], we use a form that generalizes Eq. (4):

$$C_{ag}(t,t_w) \simeq \mathcal{C}[h(t)/h(t_w)]. \tag{8}$$

A widely used form for $h(u)$, which we choose here, is $h(u)=\exp[u^{1-\mu}/(1-\mu)]$ where μ allows us to interpolate between superaging ($\mu>1$) and subaging ($\mu<1$) via simple aging ($\mu=1$). In Fig. 5(a), we show that this form with $\mu=1.035$ allows for a nice collapse of the curves presented in Fig. 1 for different t_w, corresponding to $p=0.75$ and $T=0.7T_c$. We point out that a good data collapse is also obtained (with the same exponent μ) when $C_{st}(t)$ is not substracted. In Fig. 5(b) we show a plot of $C(t,t_w)$ as a function of $h(t)/h(t_w)$ for $p=0.75$ and $T=0.5T_c$. For this temperature, the best data collapse is obtained for a larger value of $\mu=1.042$, which suggests that μ is a decreasing function of T (and one expects $\mu\to 1$ for $T\to T_c$). We would like to emphasize that the two-times scaling used in Fig. 5 is different from $C_{ag}(t,t_w)\equiv \mathcal{C}_\nu(t/t_w^\nu)$ with $\nu>1$, but corresponds to the superposition of infinitely many terms of the form $\mathcal{C}_\nu(t/t_w^\nu)$ with some distribution of the exponent $\nu>1$ [21] and is thus a feasible scaling form.

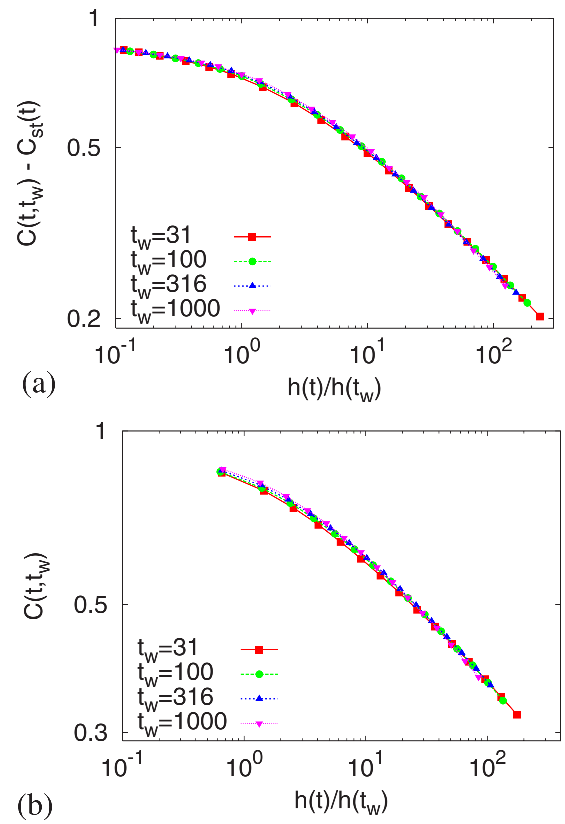

FIG. 5. (Color online) (a) $C_{\rm ag}(t,t_w)$ plotted as a function of $h(t)/h(t_w)$ with $h(t)=\exp[t^{1-\mu}/(1-\mu)]$ for different $t_w=31$, 100, 316, and 1000, with $\mu=1.035$. Data set is obtained for $p=0.75$ and $T=0.7T_c$. (b) $C(t,t_w)$ plotted as a function of $h(t)/h(t_w)$ with $\mu=1.042$. Data set is obtained for $p=0.75$ and $T=0.5T_c$.

In Ref. [22], such a superaging behavior—with comparable values of μ—was also observed in the 4d Edwards Anderson spin glass. There it was argued that superaging is consistent with a growth law $t(L)$ of the form

$$t(L) \simeq \tau_0 L^{z_c} \exp[\Upsilon(T) L^{\psi}/T], \tag{9}$$

where z_c is the dynamic critical exponent [and here $z_c=2.1667(5)$ [23]], ψ is the barrier exponent, and $\Upsilon(T)$ a typical free-energy scale (vanishing at T_c). If one assumes $h(t)=L(t)$ in Eq. (8) with $t(L)$ as in Eq. (9) then one can identify $\psi/z_c=(\mu-1)$ [22]. In our case this would give a T-dependent barrier exponent ψ (see Fig. 5). In addition, the values obtained for ψ from that relation are different from the exact value $\psi=1/4$ [5].

To conclude, we have performed a detailed numerical study of the autocorrelation function during the coarsening dynamics of diluted Ising ferromagnets in dimension $d=2$. Our data show clear deviations from a simple t/t_w scaling, which were also observed in a recent work on a random ferromagnet in $d=2$ [6]. However, attempts to fit the data to the simple scaling form as in Eq. (5) leads, as in Ref. [6], to $x<0$, which is unphysical. Here we proposed an alternative way of describing the dynamics in terms of superaging: this allows for a consistent description of the autocorrelation function in disordered ferromagnets. If our results reflect correctly the asymptotic scaling behavior of the autocorrelation in two-dimensional disordered ferromagnets one would thus conclude that it is not accurately described by local scale invariance as in Ref. [24]. With regard to a recent experimental study of superaging in spin glasses [25], it would be interesting to understand whether the superaging behavior we find is related to the choice of the initial conditions for the dynamics.

G.S. acknowledges the financial support provided through the European Community's Human Potential Program under Contract No. HPRN-CT-2002-00307, DYGLAGEMEM. G.S. and R.P. thank A. Coniglio, M. Henkel, and M. Pleimling for useful correspondence.

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