Random Quantum Magnets with Long-Range Correlated Disorder: Enhancement of Critical and Griffiths-McCoy Singularities

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We study the effect of spatial correlations on quenched disorder in random quantum magnets at and near a quantum critical point. In random transverse-field Ising systems disorder correlations that decay algebraically with an exponent ρ change the universality class of the transition for small enough ρ, and off-critical Griffiths-McCoy singularities are enhanced. We present exact results for 1D utilizing a mapping to fractional Brownian motion and generalize the predictions for the critical exponents and the generalized dynamical exponent in the Griffiths phase to $d \geq 2$.

The presence of quenched disorder is known to have a pronounced effect on quantum (or zero-temperature) phase transitions: new universality classes emerge, some of them with unconventional scaling properties, quantum activated dynamics, and strong, so called Griffiths-McCoy (GM) singularities [1,2] even away from the critical point. A well studied example is the random transverse Ising model, for which many results in one dimension exist [3–5]. Many of the unusual properties in 1D were recently shown to persist also in higher dimensions [6–8]. In this paper we investigate the effect of long-range correlated disorder on the critical and off-critical singularities at quantum phase transitions. This issue has, to our knowledge, never been studied before, although it is known to have significant impact on various other physical phenomena like classical (thermal) phase transitions [9], surface properties [10,11], anomalous diffusion in disordered environments [12], and many other areas.

In addition to the experimental relevance for the quantum Ising spin glasses [13] an interesting relation of the random transverse Ising models to the non-Fermi liquid behavior of f-electron compounds has recently been revealed [14]. For this system the low temperature properties of the interacting Kondo impurities have been mapped onto an effective quantum spin-1/2 system with strong Ising anisotropy and random bond and transverse fields, which turn out to possess long-range spatial correlations that decay like a power law with distance [14,15]. This observation, in addition to the above mentioned general interest, motivates the study of the present paper, which investigates the effect of long-range spatial correlations in the quenched disorder on the quantum phase transition as well as on the GM singularities.

To be concrete we consider the Ising ferromagnet with transverse fields

$$H = -\sum_{\langle ij \rangle} J_{ij} \sigma_i^x \sigma_j^x - \sum_i h_i \sigma_i^z, \qquad (1)$$

where $\sigma_i^{x,z}$ are spin-1/2 operators and the interactions J_{ij} (≥ 0) and/or the transverse fields h_i (≥ 0) are quenched random variables. The spins are located on the sites of a d-dimensional lattice and $\langle ij \rangle$ denote nearest neighbor pairs on this lattice (modeling short-range interactions).

At zero temperature ($T = 0$) this model has, in any dimension d a quantum phase transition from a paramagnetic to a ferromagnetic phase at some critical value of the average ratio between bonds and fields $[\ln(h_i/J_{ij})]_{\rm av}$. Here and in the following $[\cdots]_{\rm av}$ denotes the disorder average. The distance from this critical point is δ, such that for $\delta > 0$ the system (1) is paramagnetic, and for $\delta < 0$ it is ferromagnetic. We can introduce local deviations from the critical point by introducing variables $\delta(\mathbf{r})$, which in 1D are simply given by $\delta(r) = \ln(h_r/J_{r,r+1})$. Thus one can discriminate regions in space that tend *locally* to be ferromagnetic (paramagnetic) even for $\delta > 0$ ($\delta < 0$).

Here we study systematically the effect of (isotropic) spatial correlations in the disorder that can be modeled with a disorder correlator $G(\mathbf{r})$:

$$[\delta(\mathbf{r})\delta(\mathbf{r}')]_{\rm av} = G(\mathbf{r} - \mathbf{r}'). \qquad (2)$$

Uncorrelated disorder is described by choosing $G(\mathbf{r})$ to be a delta function. The Harris criterion for correlated disorder [9,16] shows that any disorder correlator that falls off faster than $r^{-2/\nu}$ [i.e., $G(\mathbf{r}) \sim \mathcal{O}(r^{-\rho})$ with $\rho > 2/\nu$, where ν is the correlation length exponent for uncorrelated disorder] does not change the universality class of the quantum critical point of model (1) with uncorrelated disorder. On the other hand for

$$G(\mathbf{r}) \sim r^{-\rho} \quad \text{with } 0 < \rho \leq 2/\nu \quad (\leq d), \qquad (3)$$

where the last inequality holds generally for the disordered system with uncorrelated disorder [17] (cf. [18]), the disorder correlations are relevant (and thus truly long ranged), the critical exponents become different from the uncorrelated case and the quantum critical point constitutes a new universality class, which we explore in this paper.

First we consider the 1D case for which we can derive most of our results in a rigorous way utilizing a mapping of the problem to fractional Brownian motion [19,20]. First we note that $\nu = 2$ for uncorrelated disorder [3,5], which means that the long-range correlations are relevant for $\rho < 1$. The critical exponents and scaling relations can be determined by studying the finite size scaling behavior of model (1). As is shown in [5] the gap ΔE (lowest excitation energy) of a chain of length L is given by $\Delta E \approx m_s \overline{m}_s h_L \prod_{i=1}^{L-1} h_i/J_{i,i+1}$, where m_s and $\overline{m}_s$ are the left and right surface magnetizations [$\sim \mathcal{O}(1)$], respectively. Thus,

$$\ln\Delta E \propto \sum_{i=1}^{L-1} \delta(i). \tag{4}$$

Since $\delta(i)$ are random variables with zero mean at criticality that are correlated according to (2) we conclude that $\ln\Delta E$ scales like the transverse fluctuations of a correlated random walk. Since $[\{\sum_{i=1}^{L-1} \delta(i)\}^2]_{av} \sim L \int_1^L dr\, G(r) \approx L^{2-\rho}$ we have for $\rho \leq 1$

$$\ln\Delta E \propto L^{\psi(\rho)} \quad \text{with } \psi(\rho) = 1 - \rho/2. \tag{5}$$

Thus for long-range-correlated disorder the quantum activated dynamics scenario at the critical point is even enhanced and we get a new critical exponent ψ. Note that for $\rho \geq 1$ one gets $\psi = 1/2$, the result for uncorrelated disorder [3,5]. We checked this result by computing numerically the probability distribution $P_L(\Delta E)$ (see [5] for details), which we indeed found to scale like $P_L(\ln\Delta E) \sim L^{-\psi(\rho)} \tilde{p}(\ln\Delta E/L^{\psi(\rho)})$ with $\psi(\rho)$ as in (5).

The surface magnetization $m_s = \langle \sigma_1^x \rangle$ of a finite chain of length L (with the spin at site L being fixed) is given by $m_s = \{1 + \sum_{k=1}^{L-1} \prod_{i=1}^{k} (h_i/J_{i,i+1})^2\}^{-1/2}$ [5]. From this expression and (5) one sees that $[\ln m_s(L)]_{av} \sim -L^{\psi(\rho)}$, i.e., that the *typical* magnetization decays with a stretched exponential. Moreover, away from the critical point ($\delta > 0$) one has $[\ln m_s(L)]_{av} \sim -L\delta$ implying $[m_s(L,\delta)]_{typ} \propto \exp(-L/\xi_{typ})$, where we defined *typical* correlation length that is seen to scale like $\xi_{typ} \sim \delta^{-\nu_{typ}}$ with $\nu_{typ} = 1$ independent of the correlation exponent ρ.

On the other hand the *average* surface magnetization can be shown [5,21] to scale like the survival probability $P_{surv}(L)$ of a random walk of L steps. This can be related to the first return time (frt) distribution $P_{frt}(L)$ of a fractional Brownian motion [with Hurst exponent $H = 1 - \rho/2 \in [1/2, 1]$; cf. (4) and (5)], which has been shown to scale like L^{H-2} [22]. Since $P_{surv}(L) = \int_L^{\infty} P_{frt}(L) \sim L^{H-1}$ we get for $\rho \leq 1$

$$[m_s(L)]_{av} \sim L^{-x_s(\rho)} \quad \text{with } x_s(\rho) = \rho/2. \tag{6}$$

For $\rho \geq 1$ one has the known result $x_s = \beta_s/\nu = 1/2$.

From the analogy to fractional Brownian motion one can also derive the exponent ν describing the divergence of the *average* correlation length when approaching the critical point, $\xi \sim |\delta|^{-\nu}$. A nonvanishing distance from the critical point implies that the disorder configurations

are such that they give rise to a nonvanishing average for the step width $[\delta(i)]_{av} = \delta$; i.e., the fractional Brownian motion is *biased*. For $\delta > 0$ (in the paramagnetic phase) the return time distribution has an exponential cutoff beyond a characteristic length scale ξ that scales like $\delta^{1/H-1}$ [22], which yields for $\rho \leq 1$

$$\xi_{av} \sim \delta^{-\nu(\rho)} \quad \text{with } \nu(\rho) = 2/\rho. \tag{7}$$

From (6) and (7) one gets $\beta_s(\rho) = 1$, independent of ρ. For $\rho \geq 1$ it is $\nu = 2$. The finite size scaling behavior of the average surface magnetization is then described by $[m_s]_{av} = L^{-x_s} \tilde{m}(L^{1/\nu}\delta)$. In Fig. 1 we show a corresponding scaling plot for $\rho = 0.75$.

The bulk magnetization $m_b = \langle \sigma_{L/2}^x \rangle$ of such a chain is much harder to calculate; see [5]. The size dependence of the average bulk magnetization at criticality determines the last and remaining critical exponent x_b via

$$[m_b(L)]_{av} \sim L^{-x_b(\rho)}. \tag{8}$$

In the case of uncorrelated disorder it was possible to predict the exact bulk magnetization exponent $x_b = \beta_b/\nu = (3 - \sqrt{5})/4$ (that will also hold for any $\rho \geq 1$) using a particular real space renormalization group [3], which, however, appears to be inappropriate for long-range correlated disorder. In the limit $\rho \to 0$ the difference between bulk and surface magnetization will vanish due to the extreme correlation of the disorder. Hence we expect $x_b(\rho) \approx x_s(\rho) = \rho/2$ for $\rho \to 0$. To obtain the full ρ dependence we computed numerically the average magnetization for finite systems using the
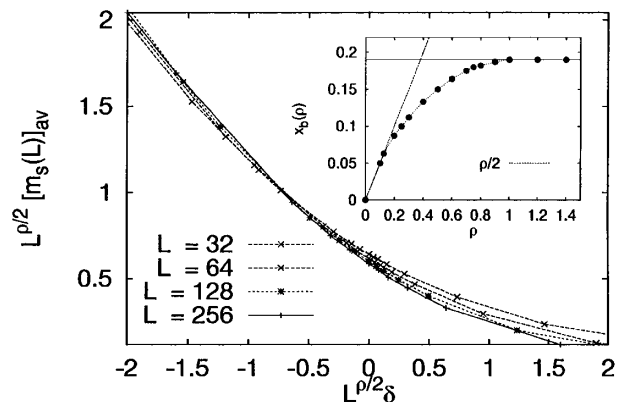

FIG. 1. Finite size scaling plot of the surface magnetization according to the form $[m_s]_{av} = L^{-x_s} \tilde{m}(L^{1/\nu}\delta)$ for $\rho = 0.75$ using (7) and (6). The data are averaged over 50 000 samples using a symmetric binary distribution for the couplings ($J_{i,i+1} \in \{\lambda, \lambda^{-1}\}$, where $\lambda = 5$, $h_i = 1$). Inset: The bulk magnetization exponent x_b as a function of the value of disorder correlation exponent ρ estimated by evaluating numerically the average persistence exponent for the same type of correlated disorder in the Sinai model (see text). The horizontal line is the value for uncorrelated disorder $x_b = (3 - \sqrt{5})/4 = 0.190\,98\ldots$; the dashed line is $\rho/2$ and represents the asymptotic dependence of $x_b(\rho)$ for $\rho \to 0$.

fermion representation [5] and analyzed its finite size scaling behavior. We found only a slight decrease of $x_b(\rho)$ with ρ, which presumably does not reflect the true asymptotic behavior that is much harder to reach numerically for correlated disorder than for uncorrelated disorder. Therefore we used the relation between the scaling properties of the bulk magnetization and the average persistence of a Sinai walker [21,23] to compute the exponent $x_b(\rho)$, as shown in the inset in Fig. 1. It confirms the asymptotic behavior $x_b(\rho) \sim \rho/2$ and the expected inequality $x_b(\rho) \leq x_s(\rho)$ [23]. In passing we note that the order parameter profiles $m(r) = [\langle \sigma_r^x \rangle]_{\mathrm{av}}$ do not have the simple scaling properties reported in [24] for the uncorrelated case.

Away from the critical point the physical properties are controlled by strongly coupled clusters (i.e., segments that have locally a tendency to order ferromagnetically [5]) giving rise to the so called GM singularities. In the paramagnetic phase ($\delta > 0$) the probability $P_L(l)$ for a strongly coupled cluster of length l is proportional to $Le^{-l/\xi}$ implying a typical length of such a cluster $l_{\mathrm{typ}} \sim \xi \ln L$, with ξ given by (7). With Eq. (4) for the gap we therefore get a typical energy scale of $\ln \Delta E \propto \delta l_{\mathrm{typ}} \propto -\delta^{1-\nu(\rho)} \ln L$. Therefore we obtain for $\rho \leq 1$ and $\delta \ll 1$

$$\Delta E \sim L^{-z'(\delta,\rho)} \quad \text{with } z'(\delta,\rho) \propto \delta^{1-2/\rho} \qquad (9)$$

[and $z'(\rho,\delta) = 2\delta^{-1}$ for $\rho \geq 1$, $\delta \ll 1$]. The generalized dynamical exponent $z'(\rho,\delta)$ parametrizes all singularities occurring in the GM phase [3,5,25]: e.g., the spin autocorrelation function at $T = 0$ decays algebraically with z', $G_{\mathrm{loc}}(\tau) = [\langle \sigma_i^x(\tau)\sigma_i(0)\rangle]_{\mathrm{av}} \sim \tau^{-1/z'}$; the local susceptibility diverges for $T \to 0$ when $z' > 1$, $\chi_{\mathrm{loc}} \sim T^{1/z'-1}$; the specific heat has an algebraic singularity at $T = 0$, $C \sim T^{1/z'}$; the magnetization in the presence of an external longitudinal field (in the x direction) scales as $M \sim H^{1/z'}$, etc. We computed numerically [5] the probability distribution $P_L(\Delta E)$ which we confirmed to have a power law tail $P_L(\Delta E) \propto \Delta E^{-1/z'+1}$ with z' as in (9).

Concluding the 1D case we stress that (for fixed distance δ from the critical point) z' increases monotonically with decreasing δ; i.e., stronger disorder correlations generate stronger GM singularities. Because of the nature of these singularities this tendency is a direct consequence of an increasing probability for large clusters for increasing disorder correlations.

Before we proceed to the higher dimensional case ($d \geq 2$) we describe briefly the infinite randomness disorder fixed point (IRFP) scenario, originally developed for uncorrelated disorder [26], but one can generalize it for the present correlated case. This phenomenological theory involves three exponents: the lowest energy scale ΔE and the linear size L of a strongly coupled cluster are related via $\ln \Delta E \sim L^\psi$, its magnetic moment scales as $\mu \sim L^{\phi\psi}$, and its typical size when approaching the critical point, the correlation length, will diverge like $\xi \sim |\delta|^{-\nu}$. All bulk exponents can be expressed via ψ,

ϕ, and ν; cf. $x_b = d - \phi\psi$, $\nu_{\mathrm{typ}} = \nu(1-\psi)$, and in the Griffiths phase $z' \propto \delta^{-\nu\psi}$. For the 1D case, as treated above, it is $\psi = 1/2$, $\phi = (\sqrt{5}+1)/2$, and $\nu = 2$ for uncorrelated disorder and $\rho > 1$, whereas for $\rho < 1$ we obtained $\psi = 1 - \rho/2$, $\phi = (1 - x_b)/\psi$ (see Fig. 1), and $\nu = 2/\rho$. The exponent relations are satisfied for the correlated and uncorrelated cases.

In higher dimensions the IRFP scenario still holds [8,26], irrespective of the presence or absence of disorder correlations. However, the exponents (ψ, ϕ, ν) will change. We can make precise statements on the change of these exponents for the case of random bond or site dilution, for which the quantum phase transition occurs at the percolation threshold $p = p_c$ (with p being the bond or site concentration). In this case the physics is completely determined by the *geometric* properties of the percolating clusters [6], which means that (ψ, ϕ, ν) can be expressed by the classical percolation exponents, which are the fractal dimension D_{perc} of the percolating cluster, the exponent β_{perc} determining the probability for a site being in the percolating cluster, and the correlation length exponent ν_{perc}, respectively. It is known [9,16] that the disorder correlations are relevant for $\rho < 2/\nu_{\mathrm{perc}}$, in which case one gets $\nu(\rho) = \nu_{\mathrm{perc}}(\rho) = 2/\rho$, $\psi(\rho) = D_{\mathrm{perc}}(\rho)$, which will increase with increasing correlations, since then clusters become more compact, and $\phi(\rho) = [d - \beta_{\mathrm{perc}}(\rho)/\nu_{\mathrm{perc}}(\rho)]/D_{\mathrm{perc}}(\rho)$, where $\beta_{\mathrm{perc}}(\rho)/\nu_{\mathrm{perc}}(\rho)$ is possibly independent of ρ [27]. In the GM phase this implies for the dynamical exponent

$$z' \sim |p - p_c(\rho)|^{-2D_{\mathrm{perc}}/\rho}, \qquad (10)$$

which increases with decreasing ρ, again confirming the general tendency that disorder correlations enhance the GM singularities.

For the generic nondiluted case the exponents need not be identical with the diluted case; however, one still has $\nu = 2/\rho$ for $\rho < 2/\nu_{\mathrm{uncorr}}$, according to a general argument given in [9]. Moreover, ψ increases with increasing disorder correlations, since its value is connected to the geometric compactness of strongly coupled clusters. Thus, the dynamical exponent $z' \sim \delta^{-\nu\psi}$ grows, again enhancing the GM singularities.

Regarding the recent experiments on f-electron systems we point out that there is evidence [15] that the spatial correlations in the metallic compound $U_{1-x}Th_xPd_3$ decay like r^{-3}. If we assume that for the transverse Ising model with uncorrelated disorder we have $\nu = 2/d$ (as is the case for $d = 1$ [3,5] and $d = 2$ [8], and also for other random quantum critical points [28,29]) such a decay with $\rho = 3$ is the marginal case and instead of modifications of the above quoted exponents strong logarithmic corrections will appear. In a scaling theory for the marginal situation one has to replace L and ξ by $L \ln L$ and $\xi \ln \xi$, respectively, yielding for $d = 3$ and $\rho = 3$ for the gap, correlation length, and dynamical exponent in the GM phase

$$\ln \Delta E \sim L^{\psi} \ln^{\psi} L,$$
$$\xi \sim \delta^{-\nu} |\ln \delta|^{-1}, \qquad (11)$$
$$z' \sim \delta^{-\nu\psi} |\ln \delta|^{\psi},$$

respectively, where ψ and ν are the critical exponents of the 3D system with *uncorrelated* disorder. Obviously these logarithmic corrections will make it very hard to extract the critical exponents ψ and ν, for instance, from experimental data, and, furthermore, will *apparently* vary when approaching the critical point.

Finally we make a few remarks on related quantum magnetic systems: (a) Quantum spin glasses [28] are expected to behave very similarly as the random ferromagnets with respect to the introduction of disorder correlations. The frustration caused by the random signs of the spin interactions is irrelevant at the IRFP [26] and therefore the universality class and the critical exponents are not changed. (b) Random *XY*, e.g., in 1D

$$H = \sum_i J^x_{i,i+1} \sigma^x_i \sigma^x_{i+1} + J^y_{i,i+1} \sigma^y_i \sigma^y_{i+1} \qquad (12)$$

and *XXZ* or Heisenberg systems have different features in 1D and in $d \geq 2$. In the latter higher dimensional case it seems that the quantum critical point is *not* an IRFP [26,30]; however, disorder correlations will certainly affect the critical properties. In 1D we encounter the same scenario as for the transverse Ising systems, in particular, for *XY* and *XX* chains [31], since these are equivalent to two decoupled transverse Ising chains. Most remarkably the transverse and longitudinal correlations still decay with the same exponent (in contrast to the pure case), however, more slowly with correlated disorder. Moreover the term *random singlet phase* is now inapproriate when $\rho < 1$, since then larger units than only pairs of spins will be strongly coupled.

To summarize we have studied the effect of long-range correlations in the disorder on the quantum critical behavior of random magnets. We have shown the relevant correlations generally enhance the critical and off-critical singularities, essentially because large strongly coupled clusters appear more frequently. For the random transverse Ising system in 1D we reported exact values for the critical exponents for arbitrary disorder correlation exponent ρ, also for the diluted case in higher dimensions and argued how these results can be generalized to generic bond/field randomness in $d \geq 2$. With respect to the recent experiments on f-electron systems we have pointed out the existence of strong logarithmic corrections that complicates the measurement of the critical and the dynamical exponenet. Finally we generalized our results to *XX* and *XY* quantum spin systems.

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