Comment on "Disorder Induced Quantum Phase Transition in Random-Exchange Spin-1/2 Chains"

In a recent Letter Hamacher, Stolze, and Wenzel [1] studied the disordered spin-1/2 antiferromagnetic XXZ chain

$$H = J \sum_{i=1}^{L-1} [\lambda_i (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + \Delta S_i^z S_{i+1}^z] \qquad (1)$$

with λ_i independent identically distributed random variables uniformly distributed over the interval $[1 - W; 1 + W]$, with the parameter W controlling the strength of the disorder. The authors claim that for $\Delta < 1$ they found numerical evidence for nonuniversal behavior for weak disorder ($W < 1$) manifested in a continuously varying exponent $\eta(W)$ describing the asymptotic decay of the transverse spin correlations

$$C^{xx}(r) = \langle S_i^x S_{i+r}^x \rangle \propto r^{-\eta(W)}, \qquad (2)$$

where $\langle \cdots \rangle$ denotes the ground state expectation value averaged over the disorder and the sites i. They concluded that there is no universal infinite randomness fixed point (IRFP) as predicted by Fisher [2].

In this Comment we show that these conclusions are inadequate because the numerical data presented in [1] are not in the asymptotic regime. We demonstrate that the observed behavior is very compatible with the IRFP scenario due to the existence of a W-dependent crossover length scale ξ_W that describes the crossover from the pure fixed point to the only relevant IRFP: For $L \ll \xi_W$ one observes the critical behavior of the pure system ($\lambda_i =$ const), and only for $L \gg \xi_W$ the true asymptotic critical behavior [$\eta(W) = 2$, independent of $W > 0$] of the disordered chain becomes visible. Even for strong disorder ($W = 1.0$) ξ_W is of the same order of magnitude as the system sizes considered in [1].

In order to be able to reach sufficiently large system sizes we restrict ourselves to $\Delta = 0$ in which case (1) reduces to a free fermion model and the ground state computations are done following [3]. In Fig. 1 we show the averaged bulk correlation function $C^{xx}(L/2)$ for different strengths of the disorder. We observe that asymptotically (i.e., for $L \to \infty$) the data follow the behavior $C^{xx}(L) \propto L^{-2}$ as predicted by the real space renormalization group [2]. Only for small L do the data *appear* to follow a nonuniversal (i.e., W-dependent) power law, and this is the region on which [1] reports.

Because of the presence of a crossover length scale $\xi(W)$, the correlation function obeys the scaling form

$$C^{xx}(L/2) = L^{-1/2} \tilde{c}(L/\xi_W), \qquad (3)$$

where $\tilde{c}(x)$const for $x \to 0$, and $\tilde{c}(x) \to x^{-3/2}$ for $x \to \infty$. This implies $C^{xx}(L/2) \propto L^{-1/2}$ for $L \ll \xi_W$ (the pure behavior) and $C^{xx}(L/2) \propto L^{-2}$ for $L \gg \xi_W$ (the IRFP behavior). In the inset of Fig. 1 we show such a scaling plot of the data in the main figure. We have chosen ξ_W for $W = 1$ such that the crossover region is centered around $L/\xi_W \approx 1$; the other estimates for ξ_W are then chosen to give the best data collapse. We see that for all disorder strengths the maximum system sizes used in [1] are still well within the crossover region and *not* in the asymptotic regime. We do not expect that this situation will improve for $\Delta > 0$. In general ξ_W diverges when $W \to 0$, and we found that our estimates for ξ_W obey $\xi_W \propto \delta_W^{-\Phi}$ where δ is the (W-dependent) variance of the random variable $\ln\lambda_i$. We obtain $\Phi \approx 1.8 \pm 0.2$.

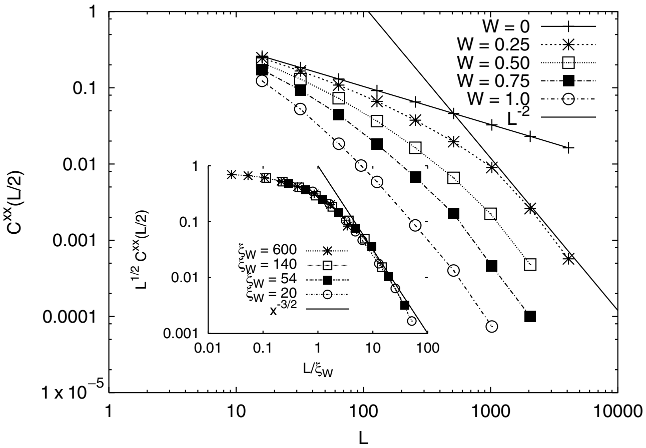

FIG. 1. Averaged correlation function $C^{xx}(L/2)$ as a function of the system size L in a log-log plot for $W = 0$, 0.25, 0.5, 0.75, and 1.0 (top to bottom). The data are averaged over 50 000 for $L \leq 1024$, 3000 for $L = 2048$, and 500 for $L = 4096$; the statistical error is smaller than the symbol sizes. The data for the pure system ($W = 0$) follow $C^{xx}(L/2) \propto L^{-1/2}$; the full line with slope -2 is the expected asymptotic behavior according to the IRFP scenario [2]. Inset: Scaling plot according to Eq. (3) for the data of the main figure with $\xi_W = 600$, 140, 54, and 20 for $W = 0.25$, 0.5, 0.75, and 1.0, respectively.

To conclude we have shown that the asymptotic behavior of model (1) belongs to the universality class described by an IRFP also for weak disorder as predicted by [2].

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Nicolas Laflorencie[1] and Heiko Rieger[2]
[1]Laboratoire de Physique Théorique, CNRS-UMR5152
Université Paul Sabatier
F-31062 Toulouse, France
[2]Theoretische Physik, Universität des Saarlandes
66041 Saarbrücken, Germany

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[1] K. Hamacher, J. Stolze, and W. Wenzel, Phys. Rev. Lett. **89**, 127202 (2002).
[2] D. S. Fisher, Phys. Rev. B **50**, 3799 (1994).
[3] F. Iglói, R. Juhász, and H. Rieger, Phys. Rev. B **61**, 11 552 (2000).

