Aging and domain growth in the two–dimensional Ising spin glass model

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Abstract

Interrupted aging in the two-dimensional Ising spin glass model with Gaussian couplings is established and investigated via extensive Monte-Carlo simulations. The spin autocorrelation function scales with $t/\tau(t_w)$, where t_w is the waiting time and τ is equal to t_w for waiting times smaller than the equilibration time $\tau_{\rm eq}$. The spatial correlations scale with $r/\xi(t_w)$, where the correlation length ξ gives information about the averaged domain size in the system. Our results are better compatible with an algebraic growth law for $\xi(t_w)$, although it can also nicely be fitted to $(\log t_w)^{1/\psi}$ with $\psi \approx 0.63$.

75.10N, 75.50L, 75.40G.

Aging is a characteristic feature of the non-equilibrium dynamics of spin glasses [1]. After the system has been quenched into a typical non-equilibrium state (for instance after decreasing the temperature rapidly) the huge relaxation times at low temperatures lead to the experimental observation of history dependent phenomena on laboratory time scales [2]. This scenario, also reported for various other strongly disordered materials [3,4], is not connected to a finite-temperature spin glass transition [6,5]. It appears that aging is just a consequence of an extremely slow relaxation and is either interrupted for time scales larger than the (possibly astronomically large) equilibration time $\tau_{\rm eq}$ or lasting forever within a spin glass phase with an infinite equilibration time.

It has been suggested [7,8] that aging is a manifestation of a slow domain growth at small temperatures, where after a certain waiting time t_w a characteristic domain size $R(t_w)$ is reached. The time dependence of subsequent observations (as for instance the thermoremanent magnetization) show a clear cross-over from dynamical processes characterized by length scales smaller than the already achieved domain size (and mainly taking place inside the domains, for which reason it is called the quasi-equilibrium regime) to processes on larger time scales dominated by the continuation of domain growth through the movement of domain walls across the system.

An activated dynamics scenario [7] is built upon the hypothesis that after a temperature-quench from a fully disordered state to low temperatures domains on a length scale L can grow by surmounting (free) energy barriers $B(L) \propto L^{\psi}$, where $\psi \leq d-1$ is some (temperature–independent) exponent. Via thermal activation such a process needs a time $\tau \propto \exp(\Delta B(L)/T)$. Thus after a waiting time t_w the characteristic domain-size is given by

$$R(t_w) \propto (\log t_w)^{1/\psi} \; . \tag{1}$$

Originally such a scenario was proposed for the dynamics of a strongly disordered system well below its phase transition temperature. In the two-dimensional Ising spin glass, which does not have a finite temperature phase transition (see e.g. [10]), this scenario cannot hold for waiting times that are comparable to the (temperature-dependent) equilibration time $\tau_{\rm eq}$.

However, for $t_w \ll \tau_{\rm eq}$ it might be valid at very low temperatures. Recent experiments on the two-dimensional short range Ising spin glass $Ru_2Cu_{0.89}Co_{0.11}F_4$ [6,9] indicated the validity of an activated dynamics scenario. However, since the domain-size or the correlation length of spatial correlations has not been measured directly (e.g. via neutron scattering), the growth-law (1) had to be verified indirectly by establishing the activated scaling behavior $|\log\omega|/\log t_w$ of the susceptibility $\chi_{t_w}(\omega)$ measured after aging the system for a waiting time t_w. From this it was then concluded that (1) should also hold.

On the other hand, in computer simulations one has immediate access to the quantities of interest and the validity of a logarithmic growth law like (1) can be checked directly. Therefore we present in this letter the results of extensive Monte-Carlo simulations of the aging scenario and the domain growth in the two-dimensional Ising spin glass model (more details will be published elsewhere [11]). The latter is defined by the Hamiltonian

$$H = -\sum_{\langle ij\rangle} J_{ij} S_i S_j \ , \tag{2}$$

where $S_i = \pm 1$, the sum extends over all nearest neighbor pairs of a $L \times L$ square lattice with periodic boundary conditions and interaction strengths J_{ij} are Gaussian random variables with zero mean and variance one. We choose the usual heat bath algorithm for the spin dynamics and time is measure in Monte-Carlo sweeps through the whole lattice. We used lattice sizes L ranging from 30 to 200 depending on the temperature T (note that one does not need to consider system sizes much larger than the typical correlation length to exclude finite size effects). The quantities of interest are strongly fluctuating from sample to sample, for which reason we averaged over several thousand disorder realizations). The simulations were performed on a Parsytec GCel1024 transputer cluster.

A straightforward way to establish aging in the present model is to calculate the spin autocorrelation function

$$C(t,t_w) = \frac{1}{N}\sum_i [\langle S_i(t+t_w) S_i(t_w)\rangle]_{\rm av} \ , \tag{3}$$

where $\langle\cdots\rangle$ means a thermal average (i.e. an average over different realizations of the thermal noise) and $[\cdots]_{\rm av}$ means an average over different realizations of the bond-disorder. For each

run the system is initialized in a random initial configuration corresponding to a quench from infinite temperature to the temperature T at which the simulation is done (e.g. $\underline{S}(t_w)$ then denotes the configuration of the system at time t_w after the quench).

In figure 1 we show the results for $C(t, t_w)$. For higher temperatures one observes that curves for waiting times t_w larger than a particular value, which we denote with $\tau_{\rm eq}$, collapse. One says that aging is interrupted here [12] and this is simply a manifestation of the fact that the system is equilibrated now and the spin correlations $[\langle S_i(t) S_i(t')\rangle]_{\rm av}$ become stationary in time (i.e. functions of $t - t'$ only). For lower temperatures equilibration is not achieved on the time-scales explored and the resulting picture is indistinguishable from that observed in the three-dimensional Ising spin glass [13] or the mean field Ising spin glass [14] within the spin glass phase. A characteristic feature of the low temperature ($T \leq 0.2$) behavior is the crossover at t_w from a very slow decay at times $t < t_w$ to a faster, algebraic decay at times $t > t_w$:

$$C(t, t_w) \propto t^{-\zeta(T)} \quad \text{for } t_w \ll \tau_{\rm eq} \ , \tag{4}$$

which, however, is expected to fail at much larger time-scales $t_w \gg \tau_{\rm eq}$. The exponent $\zeta(T)$ decreases with temperature and is e.g. 0.05 for $T = 0.2$. Although this is a rather small exponent our data exclude a logarithmic decay that is predicted for activated dynamics [7].

In figure 2 we show that $C(t, t_w)$ obeys simple dynamic scaling

$$C(t, t_w) \propto \tilde{c}[t/\tau(t_w)] \ , \tag{5}$$

where the time scale $\tau(t_w)$ is depicted in the insert. It is equal to t_w for $t_w \ll \tau_{\rm eq}$ and saturates (observable only for high enough temperatures) at $\tau_{\rm eq}$. Note that according to an activated dynamics scenario the scaling variable should be $\log(t)/\log(t_w)$ rather than t/t_w (see e.g. equation 3.20 in [7]). A t/t_w scaling has also been observed in three-dimensional spin glasses [13,15], the SK-model [14] and simplified spin glass models [12,16].

The above described aging scenario can be thought of as being a consequence of a very slow domain growth, or equivalently, the slow increase of correlated volumes in the system

after the temperature quench, as described in the introductory remarks. To verify this hypothesis we have to look at a quantity that provides us with the necessary informations. The domain growth in the various strongly disordered system like the site-diluted Ising model [17], the random field Ising model [18] and the random bond ferromagnetic Ising model [19] or the random bond Ising-chain [20] has already been investigated numerically. These models have the advantage that their ground state is known to be ferromagnetic, which makes the identification of domains easy. This is not the case for the present system — which is the reason why such an investigation is much more difficult here. One way to measure the domain size in spin glasses, which is however, very (computer-) time consuming, has been proposed in [21] by studying the three-dimensional Ising spin glass.

We decided instead to calculate an appropriate spatial correlation function

$$G(r,t_w) = \frac{1}{t_w} \sum_{t=t_w+1}^{2t_w} [\langle S_0(t) S_r(t) \rangle^2]_{\rm av} \ , \tag{6}$$

which is a generalization of the usual equilibrium correlation function $G_{\rm eq}(r) = [\{{\rm Tr}_{\underline{S}}\, S_0 S_r \exp -\beta H(\underline{S})\}^2]_{\rm av}$ to the present non-equilibrium situation (note that $\lim_{t_w\to\infty} G(r,t_w) = G_{\rm eq}(r)$). It is obvious that for small waiting times t_w the correlations decay rapidly to zero on the length scale of one lattice spacing since the initial configuration is random and spins on different sites are uncorrelated. For increasing waiting times the system tries to relax into energetically more favorable configurations (closer to the — in this case unique but unknown — ground state). This is the process of domain growth mentioned in the beginning and can be read off from (6) by an increase in the number of spins that are longer correlated (to contribute to the sum over t) over longer distances r. Thus the length scale of the decay of $G(r,t_w)$ is a measure for the averaged domain size $R(t_w)$.

Usually when calculating $G_{\rm eq}(r)$ the thermal average $\langle \cdots \rangle$ is replaced by a time average over a very long Monte-Carlo run. To get good statistics for the quantity (6), especially for small waiting times t_w, one has to average instead over many Monte-Carlo runs (the number of which decreases with increasing t_w, fortunately) using different initial conditions and different thermal noise. Furthermore one performs a spatial average by taking into

account all spin-pairs that are r sites apart in the x– as well as in the y–direction. To avoid a positive bias of the statistical errors induced by the square (which is necessary since the site correlations alternate in sign) we simulated two replicas a and b of the system simultaneously (both with identical realization of disorder, but with uncorrelated initial conditions and thermal noise) and calculated the four spin correlations $\langle S_0^a(t) S_r^a(t) S_0^b(t) S_r^b(t) \rangle$. This yields results identical to those obtained by using $\langle S_0(t) S_r(t) \rangle^2$, as we checked explicitly.

Figure 3 shows the result for $G(r, t_w)$ for different waiting times t_w at $T = 0.8$ One observes that the correlations grow steadily with increasing waiting time, until, at low enough temperature, the resulting curves collapse for $t_w > \tau_{\rm eq}$. This is in full agreement with the picture that emerged from the time-dependent autocorrelation function $C(t, t_w)$ described above. In a the linear-log plot of figure 3 the curves are only slightly bended, meaning that the decay is roughly exponential with a characteristic length scale. We define an effective correlation length (that is identical to this length scale for a pure exponential decay) via

$$\xi(t_w) = 2 \int_0^\infty dr \; G(r, t_w) \; , \tag{7}$$

which, as we mentioned above, is expected to be comparable to the averaged domain size $R(t_w)$ [22,23]. The result is depicted in figure 4, where it can be seen that $\xi(t_w)$ increases slowly on a logarithmic scale until it saturates at $\xi_{\rm eq}$ for $t_w > \tau_{\rm eq}$. We did a least square fit to a logarithmic growth law

$$\xi(t_w) - \xi_1 \propto (\log t_w)^{1/\psi} \quad \text{for } t_w \ll \tau_{\rm eq} \tag{8}$$

and obtained $\psi = 0.63 \pm 0.05$ (roughly independent of temperature), which is in agreement with the bound $\psi \leq d - 1$ and slightly smaller than the value found experimentally [6]. This is compatible with an activated dynamics scenario, see eq. (1). However, one obtains also a good result (in terms of the outcome of a chi-square test) for a fit to an algebraic growth law

$$\xi(t_w) \propto t_w^{\alpha(T)} \quad \text{for } t_w \ll \tau_{\rm eq} \, . \tag{9}$$

The exponent $\alpha(T)$ decreases with temperature, being for instance 0.08 for $T = 0.4$ and 0.04 for $T = 0.2$. These are rather small exponents, which is the reason why it is hard to discriminate between (8) and (9).

¿From our point of view an algebraic domain growth yields a picture that is more consistent when taking into account our results for the autocorrelation function. On one side the proposition (1) implies a logarithmic scaling of the autocorrelation function $C(t, t_w)$, which is not in agreement what we find, see (5), and also implies a logarithmic decay $C(t, t_w) \propto \log(t)^{-\lambda'/\psi}$ for $t \gg t_w (\ll \tau_{\rm eq})$ [7], which disagrees with our result (4). On the other hand these results (4), (5) and (9) can be explained consistently in the following way:

Let us assume a modified activated dynamics scenario for domain growth in which the (free) energy barriers $B(L)$ scale with the length scale L of the domains as $B(L) \approx a(T) \log(L)$ instead of $B(L) \propto L^{\psi}$ and let us follow [7] otherwise. Then one obtains for the domain-size $R(t_w) \propto t_w^{\alpha(T)}$, which is exactly (9) with $\alpha(T) = a(T)\Delta(T)/T$. Furthermore from $C(t, t_w) \propto [R(t_w)/R(t)]^{\lambda'}$ for $t \gg t_w (\ll \tau_{\rm eq})$ one concludes the scaling (5) as well as the algebraic time decay (4) with $\zeta(T) = \lambda' \alpha(T)$. We should mention that an algebraic growth law like (9) for the domain sizes has been used by Koper and Hilhorst as a working hypothesis in their domain theory [8].

In summary we have shown that aging is present at low temperatures even in the two-dimensional Ising spin glass model and is caused by a very slow domain growth. We have shown that the autocorrelation function scales like t/t_w and decays algebraically for waiting times smaller than the equilibration time. Fitting the time dependence of the averaged correlation length or domain size to a logarithmic growth law or to an algebraic growth law yields reasonable results in both cases. However, an algebraic fit is slightly better and is also compatible with the results for $C(t, t_w)$, whereas a logarithmic law is not. Results for the three-dimensional Ising spin glass [13] indicate that here the situation is quite similar.

We think that the simultaneous investigation of correlations in time *and* in space are very important to obtain a consistent picture of the dynamical processes in spin glasses at low temperatures. It would be very helpful if experimentalists could find a practicable

way to gain *direct* access to spatial correlation functions in spin glasses, as is, for instance, possible in random field systems [24]. In numerical simulations, however, such an endeavor is promising, as we have demonstrated here. For instance it would be of great interest to check such concepts as the overlap length in connection with temperature or field cycling experiments via a direct computation of the appropriate correlation functions in Monte-Carlo simulations (similar to those presented in this paper). That might help to resolve a long lasting debate among physicists [2,25–27].

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[23] The factor 2 on the r.h.s. in (7) is motivated by the fact that we expect that $\int_0^\infty dr \; t_w^{-1} \sum_{t=t_w+1}^{2t_w} [\,|\langle S_0(t) S_r(t)\rangle|\,]_{\rm av} \approx \xi(t_w)$, where the modulus takes care of the alternating sign of the spin correlations. Replacing the modulus by a square as in (6) reduces the integral by a factor one half in case of an exponential decay.

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FIGURES

FIG. 1. Autocorrelation function $C(t, t_w)$ as a function of time t for $t_w = 5^n$ $(n = 1, \ldots, 8)$ at $T = 1.0$ and 0.8, $(n = 2, \ldots, 8)$ at 0.6 and 0.2. The system size is $L = 100$ and the disorder average was performed over 256 samples. The errorbars are smaller than the symbols.

FIG. 2. Scaling plot of the autocorrelation function $C(t, t_w)$ versus $t/\tau(t_w)$ at $T = 0.2$, The insert shows $\tau(t_w)$.

FIG. 3. The spatial correlation function $G(r, t_w)$ for $t_w = 5^n$ $(n = 1, \ldots, 8)$ at temperatures $T = 0.8$. The insert shows a scaling plot of $G(t, t_w)$ versus $r/\tilde{\rho}(t_w)$. The scaling length $\tilde{\xi}$ is equal (within the errorbars) to $\xi(t_w)$ defined in the text and shown in the next figure. The system size is $L = 100$ and the disorder average was performed over 256 samples.

FIG. 4. Log–linear plot of the correlation length $\xi(t_w)$ as defined in the text. The full lines are least square fits to $(\log t_w)^{1/\psi}$ with $\psi = 0.66$ for $T = 0.2$, $\psi = 0.60$ for $T = 0.3$ and $\psi = 0.63$ for $T = 0.4$. The insert shows $\xi(t_w)$ at somewhat higher temperatures.