Comment on "Random Walks, Reaction-Diffusion, and Nonequilibrium Dynamics of Spin Chains in One-Dimensional Random Environments"

In a recent Letter Fisher, LeDoussal and Monthus [1] studied various properties of Sinai's model for diffusion in a one-dimensional random environment by a real space renormalization group (RSRG) calculation. It is claimed that despite its approximate character the RSRG yields asymptotically exact results. In this comment we want to show that 1) most of their results can be derived in a rigorous way without recurring to an approximate RG scheme, and 2) that in this way far more general results can be obtained, not restricted to the special case (the random force model) considered in [1], nor to the vicinity of the critical (unbiased) point.

First we would like to point out that all what follows is valid for a quite general one-dimensional random walk with nearest neighbor hopping, which is characterized by the transition probabilities $w_{i,i\pm1} = w(i \to i \pm 1)$ for a random walker to jump from site i to site $i \pm 1$. Of particular interest is the asymmetric case, $w_{i,i+1} \neq w_{i+1,i}$, for which the random force model considered in [1] is one special example.

A *new* prediction of [1] is the persistence exponent for a single walker. We would like to stress that there is an *exact* formula for the persistence probability of a single walker on a strip of width L, i.e. the probability that a walker does not return to its starting point on the left before it leaves the system on the right. It is given by [2]

$$p_{\mathrm{pr}}(L) = \left[\left(1 + \sum_{i=1}^{L} \prod_{j=1}^{i} \frac{w_{j,j-1}}{w_{j,j+1}} \right)^{-1} \right]_{\mathrm{av}} \quad (1)$$

and is completely analogous to a corresponding *exact* formula for the surface magnetization of random transverse Ising chains (RTIC) derived and analyzed in [3]. It can be shown rigorously that for the binary distribution $P(w_{i,i\pm1}) = 1/2 \cdot \{\delta(w_{i,i\pm1} - \lambda) + \delta(w_{i,i\pm1} - \lambda^{-1})\}$ in the limit $\lambda \to 0$ the above formula transforms into the survival probability of a random walker in a *homogeneous* environment with one adsorbing boundary. Thus $p_{\mathrm{pr}}(L) \propto L^{-\theta_1}$ with the persistence exponent $\theta_1 = 1/2$.

Another new prediction of [1] is the persistence properties of the thermally averaged position $\langle x(t) \rangle$, which is different from the single walker persistence. It is claimed that the probability for $\langle x(t) \rangle$ not to return to the starting value $x(0)$ before leaving a strip of width L is given by the averaged persistence probability $\overline{p}_{\mathrm{pr}}(L) \sim L^{-\overline{\theta}}$ with $\overline{\theta} = (3 - \sqrt{5})/4 = 0.19098\ldots$, related to the golden mean. This is an astonishing prediction and very much as in the case for the bulk magnetization of the RTIC [4] demands an alternative verification, since the character of the RG treatment in [1] is approximate. For this we would like to point out that for the limiting distribution mentioned above $\overline{p}_{\mathrm{pr}}(L)$ can be calculated from a random walk model in a *homogeneous* environment [5]. It is possible to determine $\overline{p}_{\mathrm{pr}}(L)$ *exactly* for strip widths up to L=14 and it turns out that usual series extrapolation methods yield an exponent

$$\overline{\theta} = 0.191 \pm 0.002 \,, \quad (2)$$

in very good agreement with the prediction of [1]. Moreover, it is possible to define a profile interpolating between the single walker and average persistence [5], analogous to the magnetization profiles in the RTIC interpolating between surface and bulk magnetization [6].

Finally in [1] the analogy between the Griffiths-McCoy phase of RTIC's and the biased case of Sinai's model has been emphasized. However, this analogy is only utilized asymptotically, i.e. for small drift close to unbiased case that corresponds to the critical point of the RTIC. We would like to stress that this correspondence goes much further, holds also deep in the Griffiths-McCoy phase and allows one to derive *new* results for the RTIC from the knowledge about anomalous diffusion properties of Sinai's model with drift. So, for instance, the dynamical exponent $z(\delta)$ (with δ being the distance from the critical point) parameterizing the Griffiths-McCoy singularities in the RTIC is given implicitly by the exact formula [5]

$$\left[\left(\frac{J}{h} \right)^{1/z(\delta)} \right]_{\mathrm{av}} = 1 \,. \quad (3)$$

where J and h are the random bonds and fields, respectively. Note that for any distribution of J and h one obtains immediately the result $1/z = 2\delta + \mathcal{O}(\delta^2)$, concurring with the RG prediction.

To conclude we have shown in this comment that many of the results of [1] can be derived in a rigorous way without recurring to an approximate RG scheme and that they are not restricted to the random force model or to the vicinity of the critical point.

Ferenc Iglói[a] and Heiko Rieger[b]

[a] Research Institute for Solid State Physics and Optics
H-1525 Budapest, P.O.Box 49, Hungary

[b] HLRZ, Forschungszentrum Jülich
52425 Jülich, Germany

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