Turbulenzen

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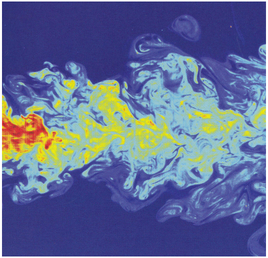
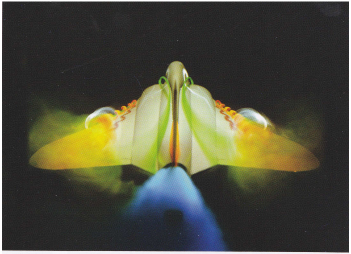
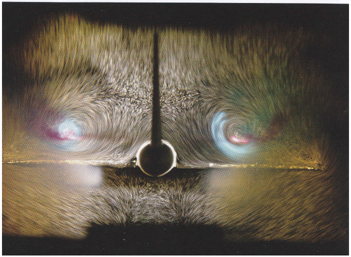
18. Mai 2011

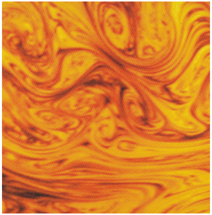


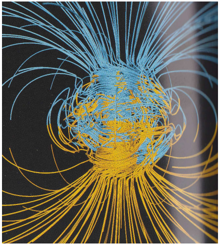
Inhalt

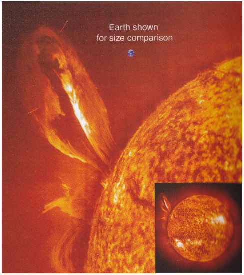
Der „natürliche Zustand“ von Fluiden







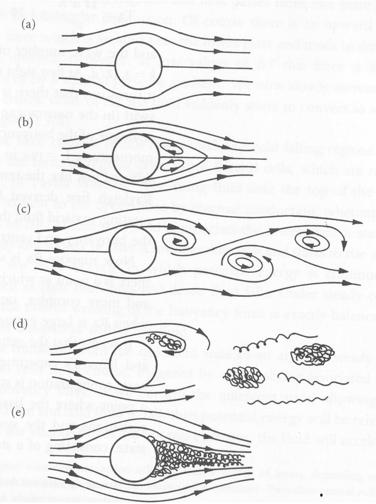




Fluss um einen Zylinder / Strömungszustand

Reynolds-Zahl $Re = \frac{ud}{\nu}$

a. $Re < 1$
b. $5 < Re < 40$
c. $100 < Re < 200$
d. $Re \sim 10^4$
e. $Re \sim 10^6$

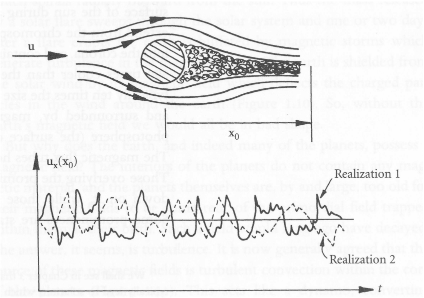


Fluss um einen Zylinder

- Turbulenzen ab einer bestimmten *Re*-Zahl beobachtbar

Turbulente Strömungen

- $\vec{u}(\vec{x}, t)$ ist zeitlich und räumlich ungeordnet
- $\vec{u}(\vec{x}, t)$ variiert von Realisierung zu Realisierung

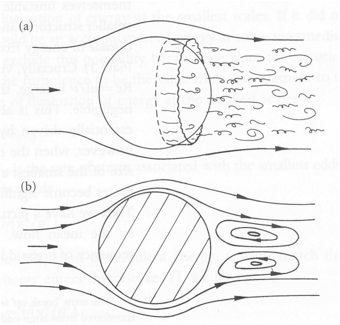


Fluss um einen Zylinder

$$\vec{u}(\vec{x}, t) = \bar{u}(\vec{x}) + \vec{u}'(\vec{x}, t)$$

Turbulente Strömungen

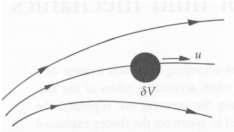
a. Schnappschuss des turbulenten Flusses
b. zeitgemittelter Fluss



Bewegungsgleichung eines Volumenelementes δV

2.Newtonsches Axiom angewendet auf δV

$$\frac{D\vec{u}}{Dt} = \frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = -\nabla\left(\frac{p}{\rho}\right) + \nu\nabla^2\vec{u}$$

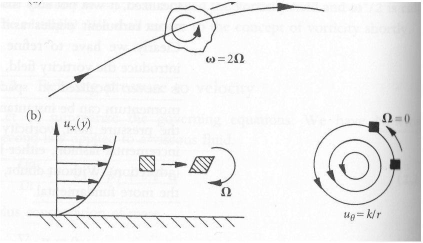


- $\vec{u}$ unterliegt einer Gleichung der Form $\frac{\partial \vec{u}}{\partial t} = F_1(\vec{u}, p)$
- für inkompressible Fluide gilt $\nabla\cdot\vec{u} = 0$
- $\nabla\cdot$ Navier-Stokes $\rightarrow \nabla^2\left(\frac{p}{\rho}\right) = -\nabla\cdot(\vec{u}\cdot\nabla\vec{u})$
- aus Biot-Savart folgt $p(\vec{x}) = \frac{\rho}{4\pi}\int\frac{[\nabla\cdot(\vec{u}\cdot\nabla\vec{u})]'}{|\vec{x}-\vec{x}'|}\mathrm{d}\vec{x}'$
- $\rightarrow \frac{\partial \vec{u}}{\partial t} = F_2(\vec{u})$

Vortex-Dynamik

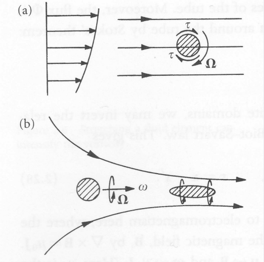
Vektorfeld $\vec{\omega} = \nabla \times \vec{u}$

- Stokes: Terminus *Winkelgeschwindigkeit des Fluidelements*
- $\vec{\omega}$ ist ein Maß für die lokale Rotation bzw. den lokalen Spin eines Fluidelements



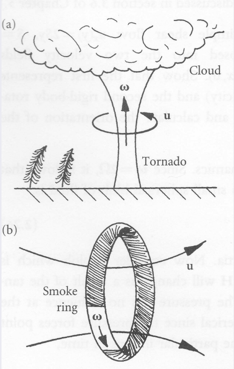
Vortex-Dynamik

$$\frac{D\vec{\omega}}{Dt} = (\vec{\omega} \cdot \nabla)\vec{u} + \nu \nabla^2 \vec{\omega}$$



Umkehr: $\vec{\omega} = \nabla \times \vec{u}$

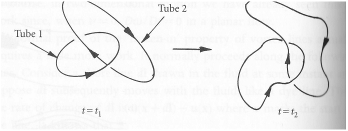
$$\vec{u}(\vec{x}) = \frac{1}{4\pi} \int \frac{\vec{\omega}(\vec{x}') \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} d\vec{x}'$$



Vortex-Dynamik

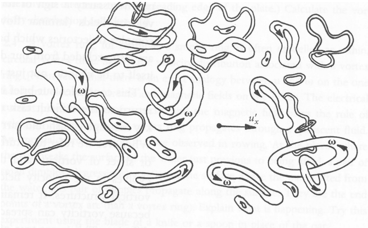
Das Theorem von Kelvin

- aus $\nu \simeq 0$ folgt $\frac{d}{dt} \int_{S_m} \vec{\omega} \cdot \mathrm{d}\vec{S} = 0$
- einzelne Vortexlinien bewegen sich mit dem Fluss



Definition:

Turbulence is a spatially complex distribution of vorticity which advects itself in a chaotic manner. The vorticity field is random in both space and time, and exhibits a wide distribution of length and time scales.



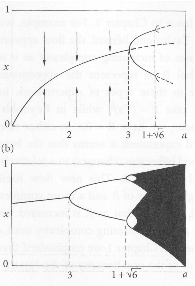
Nicht-Linearitäten und Deterministisches Chaos

$$\frac{D\vec{u}}{Dt} = \frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\triangledown)\vec{u} = -\triangledown\left(\frac{p}{\rho}\right) + \nu\,\triangledown^2\,\vec{u}$$

- ausser für kleine $\vec{u}$ sind Lösungen i.d.R. turbulent
- dafür verantwortlich: $(\vec{u}\cdot\triangledown)\vec{u}$

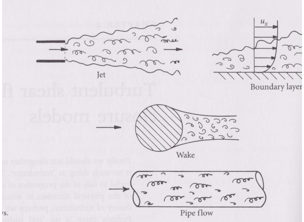
Beispiel: logistic equation (Verhulst, 1845)

- $x_{n+1} = a x_x (1 - x_n)$; $1 < a \leq 4$
- $a = 4$: $f(x) = [\pi^2 x(1-x)]^{-1/2}$

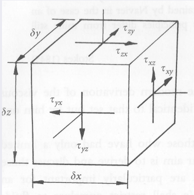


Turbulenter Scher-Fluss

- mittlere Geschwindigkeit ist hauptsächlich eindimensional



Stress-Tensor



- Newtons Gesetz der Viskosität:

$$\tau_{ij} = 2\rho\nu S_{ij} = \rho\nu\left\{\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right\}$$

Wechselwirkung zwischen mittlerem Fluss und Turbulenz

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \bigtriangledown)\vec{u} = - \bigtriangledown \left(\frac{p}{\rho}\right) + \nu \bigtriangledown^2 \vec{u}$$

- für Raumrichtung i ergibt sich:

$$\rho \frac{\partial u_i}{\partial t} + \rho(\vec{u} \cdot \bigtriangledown) u_i = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j}$$

- im zeitlichen Mittel erhält man:

$$\rho(\overline{\vec{u}} \cdot \bigtriangledown)\overline{u}_i = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\{\overline{\tau}_{ij} - \rho\overline{u_i' u_j'}\} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\{\overline{\tau}_{ij} + \tau_{ij}^R\}$$

mittlerer Impulsfluss aufgrund von Turbulenzen

$$\frac{d}{dt}\int_V (\rho \overline{u}_i)\mathrm{d}V = -\oint_S (\rho\overline{u}_i)\overline{\vec{u}} \cdot \mathrm{d}\vec{S} + \oint_S (\overline{\tau}_{ij} - \rho\overline{u_i' u_j'})\mathrm{d}S_j - \oint_S \overline{p}\mathrm{d}S_i$$

zeitliches Verhalten von $\rho \overline{u_i^{'} u_j^{'}}$

$$\frac{\overline{D}}{Dt}(\rho \overline{u_i^{'} u_j^{'}}) = ... + \frac{\partial}{\partial x_k}(-\rho \overline{u_i^{'} u_j^{'} u_k^{'}}) + ...$$

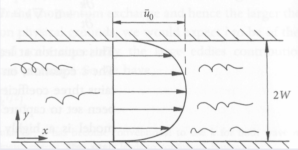
$$\frac{\overline{D}}{Dt}(\rho \overline{u_i^{'} u_j^{'} u_k^{'}}) = ... + \frac{\partial}{\partial x_m}(-\rho \overline{u_i^{'} u_j^{'} u_k^{'} u_m^{'}}) + ...$$

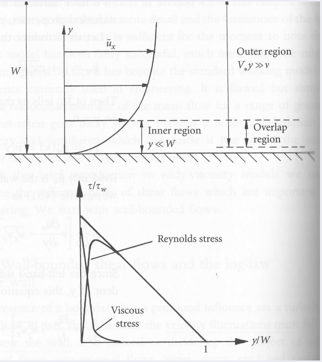
$$...$$

- also: zusätzliche Informationen vonnöten

Turbulenter Fluss durch einen Kanal

$\overline{u} = (\overline{u}_x(y), 0, 0)$





- Log-law of the wall:

$$\frac{\overline{u}_x}{V_*} = \frac{1}{0.4}\ln(V_* y/\nu) + A$$

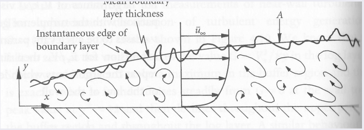
$$\frac{\overline{u}_0 - \overline{u}_x}{V_*} = \frac{-1}{0.4}\ln(y/W) + B$$

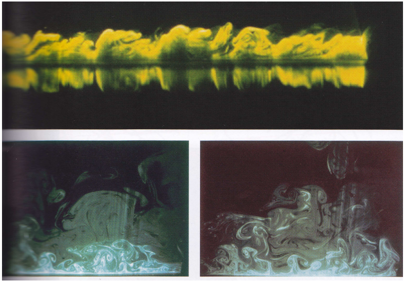
Turbulente Grenzschicht

$\overline{u} = (\overline{u}_x(y), 0, 0)$

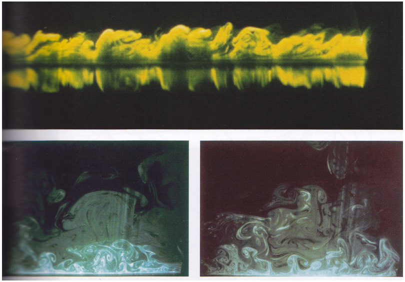
- Log-law of the wall:

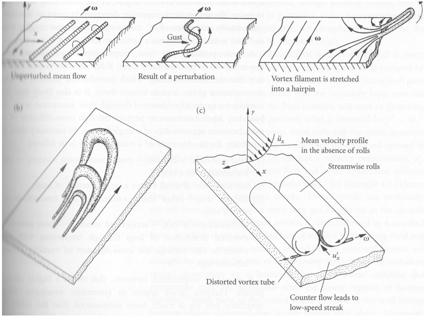
  $$\frac{\overline{u}_x}{V_*} = \frac{1}{0.4}\ln(V_* y/\nu) + A$$





Kohärente Strukturen

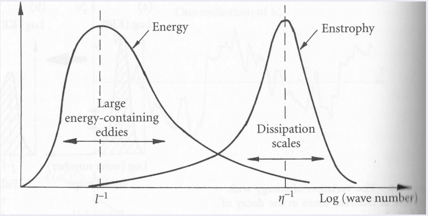




Zeit- und Längenskalen

einige Fakten zu Turbulenzen

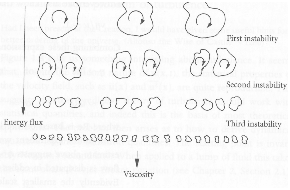
- große Bandbreite an Zeit- und Längenskalen
- Vortizität v.a. auf kleinsten Skalen
- Energiedissipation mit Rate $\varepsilon = 2\nu S_{ij} S_{ij} = \nu \vec{\omega}^2$
  $\vec{\omega}^2/2 =$ *Enstrophy*



Zeit- und Längenskalen

Energiekaskade

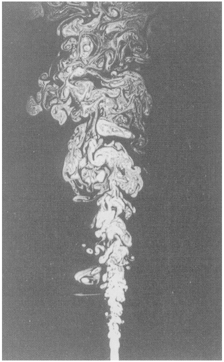
- große Skalen: $Re = ul/\nu \gg 1$
- große Skalen: Lebenszeit $\sim l/u$
  $\rightarrow \Pi \sim u^3/l$
- kleine Skalen: $Re = v\eta/\nu \sim 1$
  $\varepsilon = \nu\vec{\omega}^2 \sim \nu v^2/\eta^2$

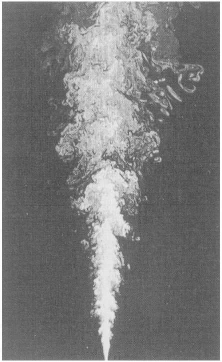


$$\Pi \sim u^3/l \sim \varepsilon \sim \nu v^2/\eta^2$$

$$\rightarrow \eta \sim l \cdot Re^{-3/4} \text{ und } v \sim u \cdot Re^{-1/4}$$

Zeit- und Längenskalen





Danke!